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THE MOTION OF A CHARGED PARTICLE IN THE ELECTROMAGNETIC FIELD OF A POLARIZATION-MODULATED WAVE

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Abstract

This article presents an exact solution of the equation of motion of a charged particle in the electromagnetic field of a high-intensity polarization-modulated wave. Expressions for the average kinetic energy of a particle without regard to its rest energy in the case of circular and linear polarization of a modulated wave are obtained. The motion of a charged particle in the field was analyzed and expressed in terms of dependences of its average kinetic energy on the electromagnetic wave intensity and on various types of modulation depths. The contribution of each type of modulation to the energy characteristics of a charged particle was demonstrated. Solving the equation of motion of a charged particle in the electromagnetic field of a plane wave opens up possibilities for various applications related, in particular, to various developments of multi-frequency lasers and laser modulation technology. This study was proposed due to the growing interest in experiments using high-intensity femtosecond laser radiation and high-temperature plasma

Keywords

Polarization, polarization modulation, charged particle, average kinetic energy, equation of motion

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Introduction. Currently, modulation of electromagnetic waves finds a wide applications in the field of science and technology. One of the applied modulations stands out from the rest. This is about polarization modulation, which creates a two-dimensional signal, which is its distinguishing feature from frequency and amplitude modulation. Its practical value is fully manifested in microscopy, emitter condition monitoring, diversity reception, interference suppression and more [1, 2]. The process of polarization modulation consists in changing the spatial orientation of the electric intensity vector of an electromagnetic wave according to a certain harmonic law. Thus, the essence of polarization modulation is the joint modulation of the amplitude and phase of an electromagnetic wave. This type of modulation provides good indicators of noise immunity and information transfer rate. Polarization modulation is carried out by rotating the plane of polarization [3] or by changing the polarization type[4], such modulation will be called polarization-optical [5]. Also, the same modulation of radiation can be performed by rotating the radiation source in space. In this case, the modulation is called spatially polarization. This type of modulation is used in laser therapy devices, which use amplitude and frequency modulation, and polarization as an additional modulation to them. As a result, laser radiation increases its efficiency of interaction with biological tissues, insofar as the required polarization is found for each molecule [6].

This article is devoted to the consideration of charged particle acceleration by an ultrashort laser pulse high order intensity of $10^{19}\ \mathrm{W/cm^2}$. The relevance of this topic also leaves no questions, since particle acceleration obtained by the interaction of short laser radiation with plasma is one of the advanced problems of laser physics [7–9]. The results obtained can also be used in practical calculations of the corresponding experiments, in the development of modulation technology, the development of multifrequency lasers and the improvement of devices of the type [10].

The problem of the dynamics of a charged particle in the field of an external electromagnetic wave has already been analyzed quite deeply, which can be evidenced by several scientific papers. The first works considered this problem from the classical point of view belong to Ya.I. Frenkel and from the quantum to D.M. Volkov, and later to L.D. Landau and E.M. Lifshitz [11–13]. The solution of the motion equation of an elementary particle in the field of a plane monochromatic wave was obtained in [14–17]. More complex special cases, when a laser emits an electromagnetic wave modulated in frequency or amplitude were analyzed in subsequent papers [18–20]. However, there is a gap in solving the problem of the dynamics of a charged particle in an external electromagnetic field when the wave creating the field has a modulated polarization.

The aim of this work is to analyze the motion and energy properties of a charged particle in the field of a polarization-modulated electromagnetic wave.

Problem statement. The article [21] presents a problem formulation of the following form. Assuming that the amplitude of the electromagnetic wave is modulated according to the harmonic law:

$$\mathbf{b}(\xi) = \mathbf{b}_{\perp 0}\left\{1+\delta_{\text{AM}}\cos\left[\omega_0\xi + \delta_{\text{PM}}\sin\left(\sigma\omega_0'\xi+\psi_0\right)+\zeta_0\right]\right\}, \qquad (1)$$

and we apply the Jacobi — Anger expansion [22], then we obtain components of the electromagnetic wave vectors:

$$E_x = H_y = b_{0x}\left(1+\delta_{\text{AM}}\sum_{l=-\infty}^{\infty} J_l(\delta_{\text{PM}})\cos\hat{\Phi}_l\right)\sum_{n=-\infty}^{\infty} J_n(\delta_{\text{FM}})\cos\bar{\Phi}_n,$$

$$E_y = -H_x = f b_{0y}\left(1+\delta_{\text{AM}}\sum_{l=-\infty}^{\infty} J_l(\delta_{\text{PM}})\cos\hat{\Phi}_l\right)\sum_{n=-\infty}^{\infty} J_n(\delta_{\text{FM}})\sin\bar{\Phi}_n, \qquad (2)$$

$$E_z = H_z = 0.$$

The z-axis is the direction of the electromagnetic wave propagation, when the x and y axes coincide with the direction of the semiaxes of the wave polarization ellipse b_{0x} и b_{0y}, moreover $b_{0x} \ge b_{0y} \ge 0$; the relativistic case corresponds to the condition $\xi = t - (z/c)$; ω is wave carrier frequency; $f = \pm 1$ is polarization parameter: the upper and lower signs in the expressions for E_y correspond to right and left polarization; δ_{AM} is amplitude modulation depth and $\delta_{\text{AM}} \in [0,1]$; ω_0 is modulation frequency; δ_{PM} is polarization modulation depth; $\sigma = \omega_1/\omega_0'$, $\sigma \in [0,1]$, is modulation coefficient; δ_{FM} is frequency modulation depth; ψ_0 is initial phase of the wave $\psi_0 \in [0, 2\pi]$; $J_n(\delta_{\text{FM}})$, $J_l(\delta_{\text{PM}})$ are n-th order and l-th order Bessel function; $\bar{\Phi}_n = \left(\omega + n\omega'\right)\xi + \alpha + n\varphi_0$; $\hat{\Phi}_l = \left(\omega + l\omega_0'\right)\xi + {}$ $+ \alpha + l\varphi_0$.

Motion of a charged particle in the field of a polarization-modulated electromagnetic wave. The equation of motion of a particle of mass m and charge q has the form:

$$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E} + \frac{1}{c}\left[\mathbf{v}\times\mathbf{H}\right]\right). \qquad (3)$$

The solution of equation (3) applying equation system (2) gives particle momentum components:

$$p_x = \frac{q b_{0x}}{\omega}\left[\sum_{n=-N}^{N}\frac{J_n\left(\delta_{\text{FM}}\right)}{1+n\hat{\alpha}}\sin\bar{\Phi}_n + \frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n\left(\delta_{\text{FM}}\right)J_l\left(\delta_{\text{PM}}\right)}{n\hat{\alpha}-l\hat{\alpha}_0}\times\right.$$

$$\left.\times\sin\left(\bar{\Phi}_n - \hat{\Phi}_l\right) + \frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n\left(\delta_{\text{FM}}\right)J_l\left(\delta_{\text{PM}}\right)}{2+n\hat{\alpha}+l\hat{\alpha}_0}\sin\left(\bar{\Phi}_n + \hat{\Phi}_l\right)\right] + \chi_x,$$

$$p_y = \mp \frac{q b_{0y}}{\omega}\left[\sum_{n=-N}^{N}\frac{J_n(\delta_{\mathrm{FM}})}{1+n\hat{\alpha}}\cos\bar{\Phi}_n + \frac{\delta_{\mathrm{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{FM})J_l(\delta_{\mathrm{PM}})}{n\hat{\alpha}-l\hat{\alpha}_0}\times\right.$$

$$\left.\times\cos\left(\bar{\Phi}_n-\hat{\Phi}_l\right)+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{\mathrm{FM}})J_l(\delta_{\mathrm{PM}})}{2+n\hat{\alpha}+l\hat{\alpha}_0}\cos\left(\bar{\Phi}_n+\hat{\Phi}_l\right)\right]+\chi_y,$$

$$p_z = \gamma g, \tag{4}$$

where $\hat{\alpha}=\omega'/\omega$; $\hat{\alpha}_0=\omega_0'/\omega$; $\gamma = mc\left(1-v_{0z}/c\right)/\sqrt{1-v_0^2/c^2}$; χ_x, χ_y are the integration constants.

Note that the expression was replaced by the component p_z, which will be used in further calculations. Thus, the value of g will have the following form:

$$g = h - \frac{q^2\left(b_{0x}^2-b_{0y}^2\right)}{4\gamma^2\omega^2}\left\{\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{(1+n\hat{\alpha})^2}\cos 2\bar{\Phi}_n+\right.$$

$$\left.+\frac{\delta_{\mathrm{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}J_n^2(\delta_{\mathrm{FM}})J_l^2(\delta_{\mathrm{PM}})\left[\frac{\cos 2\left(\bar{\Phi}_n+\hat{\Phi}_l\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}+\frac{\cos 2\left(\bar{\Phi}_n-\hat{\Phi}_l\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}\right]\right\}+$$

$$+\frac{q^2}{\gamma^2\omega^2}\left\{\frac{\delta_{\mathrm{AM}}}{2}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})J_l(\delta_{\mathrm{PM}})}{(1+n\hat{\alpha})(n\hat{\alpha}-l\hat{\alpha}_0)}\left[b_{0x}^2\sin\bar{\Phi}_n\sin\left(\bar{\Phi}_n-\hat{\Phi}_l\right)+\right.\right.$$

$$\left.+\,b_{0y}^2\cos\bar{\Phi}_n\cos\left(\bar{\Phi}_n-\hat{\Phi}_l\right)\right]+\frac{\delta_{\mathrm{AM}}}{2}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})J_l(\delta_{\mathrm{PM}})}{(1+n\hat{\alpha})\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)}\times$$

$$\times\left[b_{0x}^2\sin\bar{\Phi}_n\sin\left(\bar{\Phi}_n+\hat{\Phi}_l\right)+b_{0y}^2\cos\bar{\Phi}_n\cos\left(\bar{\Phi}_n+\hat{\Phi}_l\right)\right]+$$

$$+\frac{\delta_{\mathrm{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})J_l^2(\delta_{\mathrm{PM}})}{(n\hat{\alpha}-l\hat{\alpha}_0)\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)}\times$$

$$\left.\times\left[b_{0x}^2\sin\left(\bar{\Phi}_n-\hat{\Phi}_l\right)\sin\left(\bar{\Phi}_n+\hat{\Phi}_l\right)+b_{0y}^2\cos\left(\bar{\Phi}_n-\hat{\Phi}_l\right)\cos\left(\bar{\Phi}_n+\hat{\Phi}_l\right)\right]\right\}+$$

$$+\frac{q}{\gamma^2\omega}\left\{\sum_{n=-N}^{N}\frac{J_n(\delta_{\mathrm{FM}})}{1+n\hat{\alpha}}\left(\chi_x b_{0x}\sin\bar{\Phi}_n\mp\chi_y b_{0y}\cos\bar{\Phi}_n\right)+\right.$$

$$+\frac{\delta_{\mathrm{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{\mathrm{FM}})J_l(\delta_{\mathrm{PM}})}{2+n\hat{\alpha}+l\hat{\alpha}_0}\left[\chi_x b_{0x}\sin\left(\bar{\Phi}_n+\hat{\Phi}_l\right)\mp\chi_y b_{0y}\cos\left(\bar{\Phi}_n+\hat{\Phi}_l\right)\right]+$$

$$\left.+\frac{\delta_{\mathrm{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{\mathrm{FM}})J_l(\delta_{\mathrm{PM}})}{n\hat{\alpha}-l\hat{\alpha}_0}\left[\chi_x b_{0x}\sin\left(\bar{\Phi}_n-\hat{\Phi}_l\right)\mp\chi_y b_{0y}\cos\left(\bar{\Phi}_n-\hat{\Phi}_l\right)\right]\right\}.$$

Constant h has the form

$$h=\frac{1}{2}\left\langle \frac{m^2c^2+\chi_x^2+\chi_y^2}{\gamma^2}-1+\frac{q^2\left(b_{0x}^2+b_{0y}^2\right)}{\gamma^2\omega^2}\times\right.$$

$$\times\left\{\sum_{n=-N}^{N}\frac{J_n^2\left(\delta_{\text{FM}}\right)}{2\left(1+n\hat{\alpha}\right)^2}+\frac{\delta_{\text{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}J_n^2\left(\delta_{\text{FM}}\right)J_l^2\left(\delta_{\text{PM}}\right)\times\right.$$

$$\left.\left.\times J_n^2\left(\delta_{\text{FM}}\right)J_l^2\left(\delta_{\text{PM}}\right)\frac{\left(1+n\hat{\alpha}\right)^2+\left(1+l\hat{\alpha}_0\right)^2}{\left[\left(1+n\hat{\alpha}\right)^2-\left(1+l\hat{\alpha}_0\right)^2\right]^2}\right\}\right\rangle. \tag{5}$$

From equation system (4) we can obtain a parametric representation of the particle velocity:

$$v_x=\left(1-\frac{v_z}{c}\right)\left\langle\frac{qcb_{0x}}{\omega\gamma}\left\{\sum_{n=-N}^{N}\frac{J_n\left(\delta_{\text{FM}}\right)}{1+n\hat{\alpha}}\sin\bar{\Phi}_n+\right.\right.$$

$$\left.\left.+\frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}J_n\left(\delta_{\text{FM}}\right)J_l\left(\delta_{\text{PM}}\right)\left[\frac{\sin\left(\bar{\Phi}_n-\hat{\Phi}_l\right)}{n\hat{\alpha}-l\hat{\alpha}_0}+\frac{\sin\left(\bar{\Phi}_n+\hat{\Phi}_l\right)}{2+n\hat{\alpha}+l\hat{\alpha}_0}\right]+\frac{c}{\gamma}\chi_x\right\}\right\rangle,$$

$$v_y=\left(1-\frac{v_z}{c}\right)\left\langle\mp\frac{qcb_{0y}}{\omega\gamma}\left\{\sum_{n=-N}^{N}\frac{J_n\left(\delta_{\text{FM}}\right)}{1+n\hat{\alpha}}\cos\bar{\Phi}_n+\right.\right.$$

$$\left.\left.+\frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}J_n\left(\delta_{\text{FM}}\right)J_l\left(\delta_{\text{PM}}\right)\left[\frac{\cos\left(\bar{\Phi}_n-\hat{\Phi}_l\right)}{n\hat{\alpha}-l\hat{\alpha}_0}+\frac{\cos\left(\bar{\Phi}_n+\hat{\Phi}_l\right)}{2+n\hat{\alpha}+l\hat{\alpha}_0}\right]+\frac{c}{\gamma}\chi_y\right\}\right\rangle,$$

$$v_z=\frac{cg}{1+g}.$$

The next step is to find the coordinates of a charged particle from equation system:

$$x=x_0-\frac{qb_{0x}}{\gamma k\omega}\left\{\sum_{n=-N}^{N}\frac{J_n\left(\delta_{\text{FM}}\right)}{\left(1+n\hat{\alpha}\right)^2}\left(\cos\bar{\Phi}_n-\cos\bar{\Phi}_{0n}\right)+\right.$$

$$+\frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n\left(\delta_{\text{FM}}\right)J_l\left(\delta_{\text{PM}}\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}\left[\cos\left(\bar{\Phi}_n-\bar{\Phi}_l\right)-\cos\left(\bar{\Phi}_{0n}-\bar{\Phi}_{0l}\right)\right]+$$

$$\left.+\frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n\left(\delta_{\text{FM}}\right)J_l\left(\delta_{\text{PM}}\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\left[\cos\left(\bar{\Phi}_n+\bar{\Phi}_l\right)-\cos\left(\bar{\Phi}_{0n}+\bar{\Phi}_{0l}\right)\right]\right\}+$$

$$+\frac{c\chi_x}{\gamma}\left(\xi-\xi_0\right),$$

$$y = y_0 \mp \frac{qb_{0y}}{\gamma k\omega}\left\{\sum_{n=-N}^{N}\frac{J_n(\delta_{\text{FM}})}{(1+n\hat{\alpha})^2}\left(\sin\bar{\Phi}_n - \sin\bar{\Phi}_{0n}\right) + \right.$$

$$+\frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\rho)J_l(\eta)}{(n\hat{\alpha}-l\hat{\alpha}_0)^2}\left[\sin\left(\bar{\Phi}_n-\bar{\Phi}_l\right)-\sin\left(\bar{\Phi}_{0n}-\bar{\Phi}_{0l}\right)\right]+$$

$$\left.+\frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\rho)J_l(\eta)}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^2}\left[\sin\left(\bar{\Phi}_n+\bar{\Phi}_l\right)-\sin\left(\bar{\Phi}_{0n}+\bar{\Phi}_{0l}\right)\right]\right\}+$$

$$+\frac{c\chi_y}{\gamma}(\xi-\xi_0),$$

$$z = z_0 + ch(\xi-\xi_0) - \frac{q^2\left(b_{0x}^2-b_{0y}^2\right)}{8\gamma^2k\omega^2}\left\{\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\text{FM}})}{(1+n\hat{\alpha})^3}\left(\sin 2\bar{\Phi}_n - \sin 2\bar{\Phi}_{0n}\right)+\right.$$

$$+\frac{\delta_{\text{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2(\delta_{\text{FM}})J_l^2(\delta_{\text{PM}})}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^3}\left[\sin 2\left(\bar{\Phi}_n+\hat{\Phi}_l\right)-\sin 2\left(\bar{\Phi}_{0n}+\hat{\Phi}_{0l}\right)\right]+$$

$$\left.+\frac{\delta_{\text{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2(\delta_{\text{FM}})J_l^2(\delta_{\text{PM}})}{(n\hat{\alpha}-l\hat{\alpha}_0)^3}\left[\sin 2\left(\bar{\Phi}_n-\hat{\Phi}_l\right)-\sin 2\left(\bar{\Phi}_{0n}-\hat{\Phi}_{0l}\right)\right]\right\}+$$

$$+\frac{q^2}{2\gamma^2k\omega^2}\delta_{\text{AM}}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\text{FM}})J_l(\delta_{\text{PM}})}{2(1+n\hat{\alpha})(n\hat{\alpha}-l\hat{\alpha}_0)}\times$$

$$\times\left[\left(b_{0x}^2+b_{0y}^2\right)\frac{\sin\hat{\Phi}_l-\sin\hat{\Phi}_{0l}}{1+l\hat{\alpha}_0}-\left(b_{0x}^2-b_{0y}^2\right)\frac{\sin\left(2\bar{\Phi}_n-\hat{\Phi}_l\right)-\sin\left(2\bar{\Phi}_{0n}-\hat{\Phi}_{0l}\right)}{2(1+n\hat{\alpha})-(1+l\hat{\alpha}_0)}\right]+$$

$$+\frac{q^2}{2\gamma^2k\omega^2}\delta_{\text{AM}}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\text{FM}})J_l(\delta_{\text{PM}})}{2(1+n\hat{\alpha})(2+n\hat{\alpha}+l\hat{\alpha}_0)}\times$$

$$\times\left[\left(b_{0x}^2+b_{0y}^2\right)\frac{\sin\hat{\Phi}_l-\sin\hat{\Phi}_{0l}}{1+l\hat{\alpha}_0}-\left(b_{0x}^2-b_{0y}^2\right)\frac{\sin\left(2\bar{\Phi}_n+\hat{\Phi}_l\right)-\sin\left(2\bar{\Phi}_{0n}+\hat{\Phi}_{0l}\right)}{2(1+n\hat{\alpha})+(1+l\hat{\alpha}_0)}\right]+$$

$$+\frac{q^2}{2\gamma^2k\omega^2}\frac{\delta_{\text{AM}}^2}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2(\delta_{\text{FM}})J_l^2(\delta_{\text{PM}})}{4\left[(1+n\hat{\alpha})^2-(1+l\hat{\alpha}_0)^2\right]}\times$$

$$\times\left(\left(b_{0x}^2+b_{0y}^2\right)\frac{\sin 2\hat{\Phi}_l-\sin 2\hat{\Phi}_{0l}}{1+l\hat{\alpha}_0}-\left(b_{0x}^2-b_{0y}^2\right)\frac{\sin 2\hat{\Phi}_n-\sin 2\hat{\Phi}_{0n}}{1+n\hat{\alpha}}\right)-$$

$$-\frac{q}{\gamma^2k\omega}\left(\sum_{n=-N}^{N}\frac{J_n(\delta_{\text{FM}})}{(1+n\hat{\alpha})^2}\left[\chi_x b_{0x}\left(\cos\bar{\Phi}_n-\cos\bar{\Phi}_{0n}\right)\pm\chi_y b_{0y}\left(\sin\bar{\Phi}_n-\sin\bar{\Phi}_{0n}\right)\right]+\right.$$

$$+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\times$$
$$\times\Big[\chi_x b_{0x}\left(\cos\left(\bar{\Phi}_n+\hat{\Phi}_l\right)-\cos\left(\bar{\Phi}_{0n}+\hat{\Phi}_{0l}\right)\right)\pm$$
$$\pm\chi_y b_{0y}\left(\sin\left(\bar{\Phi}_n+\hat{\Phi}_l\right)-\sin\left(\bar{\Phi}_{0n}+\hat{\Phi}_{0l}\right)\right)\Big]+$$
$$+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}\times$$
$$\times\Big[\chi_x b_{0x}\left(\cos\left(\bar{\Phi}_n-\hat{\Phi}_l\right)-\cos\left(\bar{\Phi}_{0n}-\hat{\Phi}_{0l}\right)\right)\pm$$
$$\pm\chi_y b_{0y}\left(\sin\left(\bar{\Phi}_n-\hat{\Phi}_l\right)-\sin\left(\bar{\Phi}_{0n}-\hat{\Phi}_{0l}\right)\right)\Big]\Bigg).$$

Motion of a particle averaged over one period of oscillations. Let us rewrite the coordinates of a charged particle as functions depending on time

$$x(t)=\tilde{x}+\tilde{v}_x t+\xi(t),$$
$$y(t)=\tilde{y}+\tilde{v}_y t+\eta(t),$$
$$z(t)=\tilde{z}+\tilde{v}_z t+\zeta(t).$$

Here

$$\tilde{x}=x_0+\frac{\chi_x}{\gamma}\left(z_0-\tilde{z}\right)+\frac{qb_{0x}}{\gamma k\omega}\Bigg\{\sum_{n=-N}^{N}\frac{J_n(\delta_{FM})}{\left(1+n\hat{\alpha}\right)^2}\cos\bar{\Phi}_{0n}+$$
$$+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}J_n(\delta_{FM})J_l(\delta_{PM})\left[\frac{\cos\left(\bar{\Phi}_{0n}-\hat{\Phi}_{0l}\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}+\frac{\cos\left(\bar{\Phi}_{0n}+\hat{\Phi}_{0l}\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\right]\Bigg\};$$

$$\tilde{v}_x=\frac{c\chi_x}{\gamma}\frac{1}{1+h};$$

$$\xi(t)=-\frac{\chi_x}{\gamma}\zeta(t)-\frac{qb_{0x}}{\gamma k\omega}\Bigg\{\sum_{n=-N}^{N}\frac{J_n(\delta_{FM})}{\left(1+n\hat{\alpha}\right)^2}\cos\bar{\Phi}_n+$$
$$+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}J_n(\delta_{FM})J_l(\delta_{PM})\left[\frac{\cos\left(\bar{\Phi}_n-\hat{\Phi}_l\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}+\frac{\cos\left(\bar{\Phi}_n+\hat{\Phi}_l\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\right]\Bigg\};$$

$$\tilde{y}=y_0+\frac{\chi_y}{\gamma}\left(z_0-\tilde{z}\right)\pm\frac{qb_{0y}}{\gamma k\omega}\Bigg\{\sum_{n=-N}^{N}\frac{J_n(\delta_{FM})}{\left(1+n\hat{\alpha}\right)^2}\sin\bar{\Phi}_{0n}+$$
$$+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}J_n(\delta_{FM})J_l(\delta_{PM})\left[\frac{\sin\left(\bar{\Phi}_{0n}-\hat{\Phi}_{0l}\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}+\frac{\sin\left(\bar{\Phi}_{0n}+\hat{\Phi}_{0l}\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\right]\Bigg\};$$

$$\tilde{v}_y = \frac{c\chi_y}{\gamma}\frac{1}{1+h};$$

$$\eta(t) = -\frac{\chi_y}{\gamma}\zeta(t) \mp \frac{qb_{0y}}{\gamma k\omega}\left\{\sum_{n=-N}^{N}\frac{J_n(\delta_{\mathrm{FM}})}{(1+n\hat{\alpha})^2}\sin\bar{\Phi}_n + \right.$$

$$\left. + \frac{\delta_{\mathrm{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N} J_n(\delta_{\mathrm{FM}}) J_l(\delta_{\mathrm{PM}})\left[\frac{\sin(\bar{\Phi}_n - \hat{\Phi}_l)}{(n\hat{\alpha} - l\hat{\alpha}_0)^2} + \frac{\sin(\bar{\Phi}_n + \hat{\Phi}_l)}{(2 + n\hat{\alpha} + l\hat{\alpha}_0)^2}\right]\right\};$$

$$\tilde{z} = z_0 + \frac{q^2\left(b_{0x}^2 - b_{0y}^2\right)}{2\gamma^2 k\omega^2}\left[\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}})}{4(1+n\hat{\alpha})^3}\sin 2\bar{\Phi}_{0n} + \right.$$

$$+ \frac{\delta_{\mathrm{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}}) J_l^2(\delta_{\mathrm{PM}})}{4(2 + n\hat{\alpha} + l\hat{\alpha}_0)^3}\times$$

$$\left. \times \sin 2\left(\bar{\Phi}_{0n} + \hat{\Phi}_{0l}\right) + \frac{\delta_{\mathrm{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}}) J_l^2(\delta_{\mathrm{PM}})}{4(n\hat{\alpha} - l\hat{\alpha}_0)^3}\sin 2\left(\bar{\Phi}_{0n} - \hat{\Phi}_{0l}\right)\right] +$$

$$+ \frac{q^2}{4\gamma^2 k\omega^2}\left\{\delta_{\mathrm{AM}}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}}) J_l(\delta_{\mathrm{PM}})}{(1+n\hat{\alpha})(n\hat{\alpha} - l\hat{\alpha}_0)}\times\right.$$

$$\times\left[\left(b_{0x}^2 - b_{0y}^2\right)\frac{\sin\left(2\bar{\Phi}_{0n} - \hat{\Phi}_{0l}\right)}{2(1+n\hat{\alpha}) - (1+l\hat{\alpha}_0)} - \left(b_{0x}^2 + b_{0y}^2\right)\frac{\sin\hat{\Phi}_{0l}}{1+l\hat{\alpha}_0}\right] +$$

$$+ \delta_{\mathrm{AM}}\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}}) J_l(\delta_{\mathrm{PM}})}{(1+n\hat{\alpha})(2 + n\hat{\alpha} + l\hat{\alpha}_0)}\times$$

$$\times\left[\left(b_{0x}^2 - b_{0y}^2\right)\frac{\sin\left(2\bar{\Phi}_{0n} + \hat{\Phi}_{0l}\right)}{2(1+n\hat{\alpha}) + (1+l\hat{\alpha}_0)} - \left(b_{0x}^2 + b_{0y}^2\right)\frac{\sin\hat{\Phi}_{0l}}{1+l\hat{\alpha}_0}\right] +$$

$$+ \frac{\delta_{\mathrm{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2(\delta_{\mathrm{FM}}) J_l^2(\delta_{\mathrm{PM}})}{(1+n\hat{\alpha})^2 - (1+l\hat{\alpha}_0)^2}\times$$

$$\left. \times\left[\left(b_{0x}^2 - b_{0y}^2\right)\frac{\sin 2\hat{\Phi}_{0n}}{1+n\hat{\alpha}} - \left(b_{0x}^2 + b_{0y}^2\right)\frac{\sin 2\hat{\Phi}_{0l}}{1+l\hat{\alpha}_0}\right]\right\} -$$

$$- \frac{q}{\gamma^2 k\omega}\left\{\sum_{n=-N}^{N}\frac{J_n(\delta_{\mathrm{FM}})}{(1+n\hat{\alpha})^2}\left(\chi_x b_{0x}\cos\bar{\Phi}_{0n} \pm \chi_y b_{0y}\sin\bar{\Phi}_{0n}\right) + \right.$$

$$+ \frac{\delta_{\mathrm{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{\mathrm{FM}}) J_l(\delta_{\mathrm{PM}})}{(2 + n\hat{\alpha} + l\hat{\alpha}_0)^2}\times$$

$$\times\left[\chi_x b_{0x}\cos\left(\bar{\Phi}_{0n}+\hat{\Phi}_{0l}\right)\pm\chi_y b_{0y}\sin\left(\bar{\Phi}_{0n}+\hat{\Phi}_{0l}\right)\right]+$$

$$+\frac{\delta_{\mathrm{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n\left(\delta_{\mathrm{FM}}\right)J_l\left(\delta_{\mathrm{PM}}\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}\times$$

$$\left.\times\left[\chi_x b_{0x}\cos\left(\bar{\Phi}_{0n}-\hat{\Phi}_{0l}\right)\pm\chi_y b_{0y}\sin\left(\bar{\Phi}_{0n}-\hat{\Phi}_{0l}\right)\right]\right\};$$

$$\tilde{v}_z=\frac{ch}{1+h};$$

$$\left(1+h\right)\zeta(t)=-\frac{q^2\left(b_{0x}^2-b_{0y}^2\right)}{2\gamma^2k\omega^2}\left[\sum_{n=-N}^{N}\frac{J_n^2\left(\delta_{\mathrm{FM}}\right)}{4\left(1+n\hat{\alpha}\right)^3}\sin 2\bar{\Phi}_n+\right.$$

$$+\frac{\delta_{\mathrm{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2\left(\delta_{\mathrm{FM}}\right)J_l^2\left(\delta_{\mathrm{PM}}\right)}{4(2+n\hat{\alpha}+l\hat{\alpha}_0)^3}\sin 2\left(\bar{\Phi}_n+\hat{\Phi}_l\right)+$$

$$\left.+\frac{\delta_{\mathrm{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2\left(\delta_{\mathrm{FM}}\right)J_l^2\left(\delta_{\mathrm{PM}}\right)}{4(n\hat{\alpha}-l\hat{\alpha}_0)^3}\sin 2\left(\bar{\Phi}_n-\hat{\Phi}_l\right)\right]+$$

$$+\frac{q^2}{2\gamma^2k\omega^2}\delta_{\mathrm{AM}}\sum_{n=-N}^{N}\frac{J_n^2\left(\delta_{\mathrm{FM}}\right)J_l\left(\delta_{\mathrm{PM}}\right)}{2\left(1+n\hat{\alpha}\right)\left(n\hat{\alpha}-l\hat{\alpha}_0\right)}\times$$

$$\times\left[\left(b_{0x}^2+b_{0y}^2\right)\frac{\sin\hat{\Phi}_l}{1+l\hat{\alpha}_0}-\left(b_{0x}^2-b_{0y}^2\right)\frac{\sin\left(2\bar{\Phi}_n-\hat{\Phi}_l\right)}{2\left(1+n\hat{\alpha}\right)-\left(1+l\hat{\alpha}_0\right)}\right]+$$

$$+\frac{q^2}{2\gamma^2k\omega^2}\delta_{\mathrm{AM}}\sum_{n=-N}^{N}\frac{J_n^2\left(\delta_{\mathrm{FM}}\right)J_l\left(\delta_{\mathrm{PM}}\right)}{2\left(1+n\hat{\alpha}\right)\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)}\times$$

$$\times\left[\left(b_{0x}^2+b_{0y}^2\right)\frac{\sin\hat{\Phi}_l}{1+l\hat{\alpha}_0}-\left(b_{0x}^2-b_{0y}^2\right)\frac{\sin\left(2\bar{\Phi}_n+\hat{\Phi}_l\right)}{2\left(1+n\hat{\alpha}\right)+\left(1+l\hat{\alpha}_0\right)}\right]+$$

$$+\frac{q^2}{2\gamma^2k\omega^2}\frac{\delta_{\mathrm{AM}}^2}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2\left(\delta_{\mathrm{FM}}\right)J_l^2\left(\delta_{\mathrm{PM}}\right)}{4\left(\left(1+n\hat{\alpha}\right)^2-\left(1+l\hat{\alpha}_0\right)^2\right)}\times$$

$$\times\left[\left(b_{0x}^2+b_{0y}^2\right)\frac{\sin 2\hat{\Phi}_l}{1+l\hat{\alpha}_0}-\left(b_{0x}^2-b_{0y}^2\right)\frac{\sin 2\hat{\Phi}_n}{1+n\hat{\alpha}}\right]+$$

$$+\frac{q}{\gamma^2k\omega}\left\{\sum_{n=-N}^{N}\frac{J_n\left(\delta_{\mathrm{FM}}\right)}{\left(1+n\hat{\alpha}\right)^2}\left(\chi_x b_{0x}c\,\mathrm{os}\,\bar{\Phi}_n\pm\chi_y b_{0y}\sin\bar{\Phi}_n\right)+\right.$$

$$+\frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n\left(\delta_{\text{FM}}\right)J_l\left(\delta_{\text{PM}}\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\times$$

$$\times\left[\chi_x b_{0x}\cos\left(\overline{\Phi}_n+\hat{\Phi}_l\right)\pm\chi_y b_{0y}\sin\left(\overline{\Phi}_n+\hat{\Phi}_l\right)\right]+$$

$$+\frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n\left(\delta_{\text{FM}}\right)J_l\left(\delta_{\text{PM}}\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}\times$$

$$\left.\times\left[\chi_x b_{0x}\cos\left(\overline{\Phi}_n-\hat{\Phi}_l\right)\pm\chi_y b_{0y}\sin\left(\overline{\Phi}_n-\hat{\Phi}_l\right)\right]\right\}.$$

Now, when the particle coordinates are presented as functions depending on time, let us average the obtained expressions over one period of oscillations:

$$\overline{x}=\tilde{x}+\tilde{v}_x\left(t+\frac{\tilde{T}}{2}\right)\pm\frac{q^2c\chi_y b_{0x}b_{0y}}{2\gamma^3\omega^3\left(1+h\right)}\left[\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\text{FM}})}{\left(1+n\hat{\alpha}\right)^3}+\right.$$

$$\left.+\frac{\delta_{\text{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2(\delta_{\text{FM}})J_l^2\left(\delta_{\text{PM}}\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^3}+\frac{\delta_{\text{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2\left(\delta_{\text{FM}}\right)J_l^2\left(\delta_{\text{PM}}\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^3}\right],$$

$$\overline{y}=\tilde{y}+\tilde{v}_y\left(t+\frac{\tilde{T}}{2}\right)\mp\frac{q^2c\chi_x b_{0x}b_{0y}}{2\gamma^3\omega^3\left(1+h\right)}\left[\sum_{n=-N}^{N}\frac{J_n^2\left(\delta_{\text{FM}}\right)}{\left(1+n\hat{\alpha}\right)^3}+\right.$$

$$+\frac{\delta_{\text{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2\left(\delta_{\text{FM}}\right)J_l^2\left(\delta_{\text{PM}}\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^3}+$$

$$\left.+\frac{\delta_{\text{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2\left(\delta_{\text{FM}}\right)J_l^2\left(\delta_{\text{PM}}\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^3}\right],$$

$$\overline{z}=\tilde{z}+\tilde{v}_z\left(t+\frac{\tilde{T}}{2}\right).$$

The next step is to obtain averaged expressions for the momentum components of a charged particle:

$$\overline{p}_x=\chi_x\left\{1+\frac{q^2b_{0x}^2}{2\gamma^2\omega^2\left(1+h\right)}\left[\sum_{n=-N}^{N}\frac{J_n^2\left(\delta_{\text{FM}}\right)}{\left(1+n\hat{\alpha}\right)^2}+\right.\right.$$

$$\left.\left.+\frac{\delta_{\text{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2\left(\delta_{\text{FM}}\right)J_l^2\left(\delta_{\text{PM}}\right)}{\left(n\hat{\alpha}-l\hat{\alpha}_0\right)^2}+\frac{\delta_{\text{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^2\left(\delta_{\text{FM}}\right)J_l^2\left(\delta_{\text{PM}}\right)}{\left(2+n\hat{\alpha}+l\hat{\alpha}_0\right)^2}\right]\right\},$$

$$\overline{p}_y = \chi_y \left\{ 1 + \frac{q^2 b_{0y}^2}{2\gamma^2 \omega^2 (1+h)} \left[\sum_{n=-N}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\hat{\alpha})^2} + \right.\right.$$

$$+ \frac{\delta_{\text{AM}}^2}{4} \sum_{n=-N}^{N} \sum_{l=-N}^{N} \frac{J_n^2(\delta_{\text{FM}}) J_l^2(\delta_{\text{PM}})}{(n\hat{\alpha} - l\hat{\alpha}_0)^2} +$$

$$\left.\left. + \frac{\delta_{\text{AM}}^2}{4} \sum_{n=-N}^{N} \sum_{l=-N}^{N} \frac{J_n^2(\delta_{\text{FM}}) J_l^2(\delta_{\text{PM}})}{(2 + n\hat{\alpha} + l\hat{\alpha}_0)^2} \right] \right\},$$

$$\overline{p}_z = \frac{\gamma}{1+h} \left\langle (h + h^2) + \frac{q^4 \left(b_{0x}^2 - b_{0y}^2\right)^2}{32\gamma^4 \omega^4} \left[\sum_{n=-N}^{N} \frac{J_n^4(\delta_{\text{FM}})}{(1+n\hat{\alpha})^4} + \right.\right.$$

$$+ \frac{\delta_{\text{AM}}^4}{16} \sum_{n=-N}^{N} \sum_{l=-N}^{N} \frac{J_n^4(\delta_{\text{FM}}) J_l^4(\delta_{\text{PM}})}{(2 + n\hat{\alpha} + l\hat{\alpha}_0)^4} +$$

$$\left. + \frac{\delta_{\text{AM}}^4}{16} \sum_{n=-N}^{N} \sum_{l=-N}^{N} \frac{J_n^4(\delta_{\text{FM}}) J_l^4(\delta_{\text{PM}})}{(n\hat{\alpha} - l\hat{\alpha}_0)^4} \right] + \frac{q^4 \left(b_{0x}^4 + b_{0y}^4\right)}{16\gamma^4 \omega^4} \times$$

$$\times \left\{ \frac{\delta_{\text{AM}}^4}{4} \sum_{n=-N}^{N} \frac{J_n^4(\delta_{\text{FM}}) J_l^2(\delta_{\text{PM}})}{\left[(1+n\hat{\alpha})^2 - (1+l\hat{\alpha}_0)^2\right]^2} + \right.$$

$$\left. + 2\delta_{\text{AM}}^2 \sum_{n=-N}^{N} \frac{J_n^4(\delta_{\text{FM}}) J_l^2(\delta_{\text{PM}})}{(1+n\hat{\alpha})^2} \frac{(1+n\hat{\alpha})^2 + (1+l\hat{\alpha}_0)^2}{\left[(1+n\hat{\alpha})^2 - (1+l\hat{\alpha}_0)^2\right]^2} \right\} +$$

$$+ \frac{q^2}{2\gamma^4 \omega^2} \left(\chi_x^2 b_{0x}^2 + \chi_y^2 b_{0y}^2\right) \left\{ \sum_{n=-N}^{N} \frac{J_n^2(\delta_{\text{FM}})}{(1+n\hat{\alpha})^2} + 2\,\frac{\delta_{\text{AM}}^2}{4} \times \right.$$

$$\left.\left. \times \sum_{n=-N}^{N} \sum_{l=-N}^{N} J_n^2(\delta_{\text{FM}}) J_l^2(\delta_{\text{PM}}) \frac{(1+n\hat{\alpha})^2 + (1+l\hat{\alpha}_0)^2}{\left[(1+n\hat{\alpha})^2 - (1+l\hat{\alpha}_0)^2\right]^2} \right\} \right\rangle.$$

And finally, an expression for the average kinetic energy of a charged particle moving in the field of a polarization-modulated electromagnetic wave was obtained:

$$\overline{\varepsilon} = \frac{\gamma c}{1+h} \left\langle (1+h)^2 + \frac{q^4 \left(b_{0x}^2 - b_{0y}^2\right)^2}{32\gamma^4 \omega^4} \left[\sum_{n=-N}^{N} \frac{J_n^4(\delta_{\text{FM}})}{(1+n\hat{\alpha})^4} + \right.\right.$$

$$+\frac{\delta_{\text{AM}}^4}{16}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^4(\delta_{\text{FM}})J_l^4(\delta_{\text{PM}})}{(2+n\hat{\alpha}+l\hat{\alpha}_0)^4}+$$

$$\left.+\frac{\delta_{\text{AM}}^4}{16}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n^4(\delta_{\text{FM}})J_l^4(\delta_{\text{PM}})}{(n\hat{\alpha}-l\hat{\alpha}_0)^4}\right]+$$

$$+\frac{q^4\left(b_{0x}^4+b_{0y}^4\right)}{16\gamma^4\omega^4}\left\{\frac{\delta_{\text{AM}}^4}{4}\sum_{n=-N}^{N}\frac{J_n^4(\delta_{\text{FM}})J_l^2(\delta_{\text{PM}})}{\left[(1+n\hat{\alpha})^2-(1+l\hat{\alpha}_0)^2\right]^2}+\right.$$

$$\left.+2\delta_{\text{AM}}^2\sum_{n=-N}^{N}\frac{J_n^4(\delta_{\text{FM}})J_l^2(\delta_{\text{PM}})}{(1+n\hat{\alpha})^2}\frac{(1+n\hat{\alpha})^2+(1+l\hat{\alpha}_0)^2}{\left[(1+n\hat{\alpha})^2-(1+l\hat{\alpha}_0)^2\right]^2}\right\}+$$

$$+\frac{q^2}{2\gamma^4\omega^2}\left(\chi_x^2b_{0x}^2+\chi_y^2b_{0y}^2\right)\left\{\sum_{n=-N}^{N}\frac{J_n^2(\delta_{\text{PM}})}{(1+n\hat{\alpha})^2}+\right.$$

$$\left.\left.+2\frac{\delta_{\text{AM}}^2}{4}\sum_{n=-N}^{N}\sum_{l=-N}^{N}J_n^2(\delta_{\text{FM}})J_l^2(\delta_{\text{PM}})\frac{(1+n\hat{\alpha})^2+(1+l\hat{\alpha}_0)^2}{\left[(1+n\hat{\alpha})^2-(1+l\hat{\alpha}_0)^2\right]^2}\right\}\right\rangle.$$

Average kinetic energy of a particle moving in the field of a polarization-modulated electromagnetic wave of circular and linear polarization in the absence of the initial particle velocity. Let us consider the values of the average kinetic energy of a particle moving in the field of a polarization-modulated wave in circular and linear polarization in the absence of the initial particle velocity:

$$\mathbf{v}_0=0,\ \bar{\Phi}_n(0)=\bar{\Phi}_{0n}=-\omega(1+n\hat{\alpha})z_0/c,$$

$$\hat{\Phi}_l(0)=\hat{\Phi}_{0l}=-\omega(1+l\hat{\alpha}_0)z_0/c,$$

then constant χ_x, χ_y has this form:

$$\chi_x=-\frac{qb_{0x}}{\omega}\left[\sum_{n=-N}^{N}\frac{J_n(\delta_{\text{FM}})}{1+n\hat{\alpha}}\sin\bar{\Phi}_{0n}+\right.$$

$$+\frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{\text{FM}})J_l(\delta_{\text{PM}})}{n\hat{\alpha}-l\hat{\alpha}_0}\sin\left(\bar{\Phi}_{0n}-\hat{\Phi}_{0l}\right)+ \quad (6)$$

$$\left.+\frac{\delta_{\text{AM}}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{\text{FM}})J_l(\delta_{\text{PM}})}{2+n\hat{\alpha}+l\hat{\alpha}_0}\sin\left(\bar{\Phi}_{0n}+\hat{\Phi}_{0l}\right)\right],$$

$$\chi_y = \pm \frac{qb_{0y}}{\omega}\left[\sum_{n=-N}^{N} \frac{J_n(\delta_{FM})}{1+n\hat{\alpha}} \cos \bar{\Phi}_{0n} + \right.$$
$$+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{n\hat{\alpha}-l\hat{\alpha}_0}\cos(\bar{\Phi}_{0n}-\hat{\Phi}_{0l})+ \tag{6}$$
$$\left.+\frac{\delta_{AM}}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}\frac{J_n(\delta_{FM})J_l(\delta_{PM})}{2+n\hat{\alpha}\ +\ l\hat{\alpha}_0}\cos\left(\bar{\Phi}_{0n}+\hat{\Phi}_{0l}\right)\right].$$

Substituting constants (6) into equation (5), the constant takes the following form:

$$h = \frac{1}{4}\frac{q^2\left(b_{0x}^2+b_{0y}^2\right)}{\gamma^2\omega^2}\left\{\sum_{n=-N}^{N}\frac{J_n^2(\delta_{FM})}{(1+n\hat{\alpha})^2}+\right.$$
$$\left.+\frac{\delta_{AM}^2}{2}\sum_{n=-N}^{N}\sum_{l=-N}^{N}J_n^2(\delta_{FM})J_l^2(\delta_{PM})\frac{(1+n\hat{\alpha})^2+(1+l\hat{\alpha}_0)^2}{\left[(1+n\hat{\alpha})^2-(1+l\hat{\alpha}_0)^2\right]^2}\right\}.$$

Having obtained all necessary constants, let us consider the case when the particle moves in the field of a polarization-modulated electromagnetic wave of circular polarization.

For circular polarization have $b_{0x}=b_{0y}=b/\sqrt{2}$, and

$$\Psi = mc^2\frac{\mu}{2}\left\langle\frac{32\theta(1+\theta\mu)+16\mu\theta^2}{16(1+\theta\mu)}+\frac{\mu}{16(1+\theta\mu)}\times\right.$$
$$\times\left\{\frac{\delta_{AM}^4}{4}\sum_{\substack{n=-N\\ l=-N}}^{N}\frac{J_n^4(\delta_{FM})J_l^2(\delta_{PM})}{\left[(1+n\hat{\alpha})^2-(1+l\hat{\alpha}_0)^2\right]^2}+\right.$$
$$\left.\left.+2\delta_{AM}^2\sum_{\substack{n=-N\\ l=-N}}^{N}\frac{J_n^4(\delta_{FM})J_l^2(\delta_{PM})}{(1+n\hat{\alpha})^2}\frac{(1+n\hat{\alpha})^2+(1+l\hat{\alpha}_0)^2}{\left[(1+n\hat{\alpha})^2-(1+l\hat{\alpha}_0)^2\right]^2}\right\}\right\rangle, \tag{7}$$

where $\Psi = \bar{\varepsilon} - mc^2$ is the energy of a particle without considering its rest energy; $\mu = q^2b^2/(\gamma^2\omega^2) = 2q^2I\lambda^2/(\pi m^2c^5)$.

The next step is to consider the case when the particle moves in the field of a polarization-modulated electromagnetic wave of linear polarization. For linear polarization have $b_{0x}=b$, $b_{0y}=0$, and

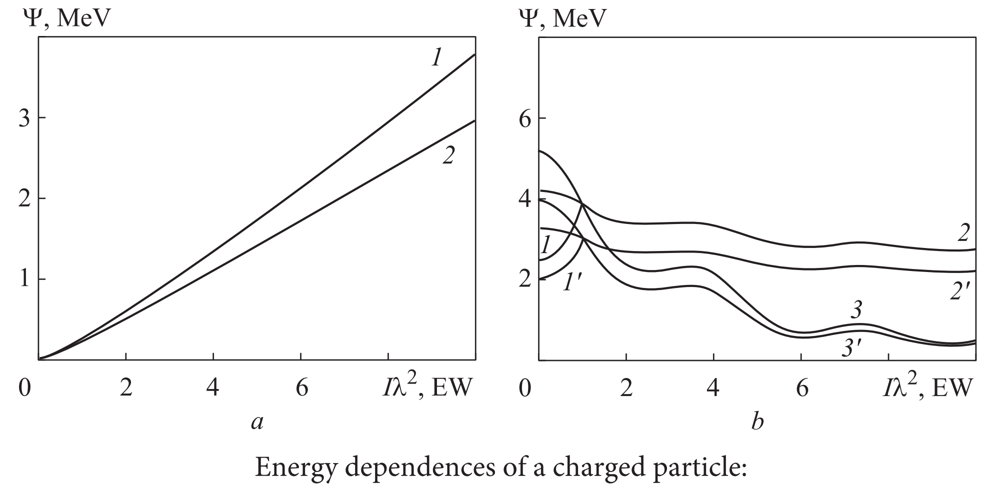
$$\Psi =$$
$$= mc^2 \frac{\mu}{4}\left\langle 12\theta + \frac{\mu T}{2\sqrt{2\mu\theta+4}\sqrt{6\mu\theta+4}} + \frac{\mu M}{\sqrt{2\mu\theta+4}\sqrt{6\mu\theta+4}} - \frac{16\theta(\mu\theta+2)}{\sqrt{2\mu\theta+4}\sqrt{6\mu\theta+4}}\right\rangle. \quad (8)$$

Here

$$\theta = \sum_{n=-N}^{N} \frac{J_n^2(\delta_{\mathrm{FM}})}{2(1+n\hat{\alpha})^2} +$$
$$+\frac{\delta_{\mathrm{AM}}^2}{4} \sum_{n=-N}^{N} \sum_{l=-N}^{N} J_n^2(\delta_{\mathrm{FM}}) J_l^2(\delta_{\mathrm{PM}}) \frac{(1+n\hat{\alpha})^2 + (1+l\hat{\alpha}_0)^2}{\left[(1+n\hat{\alpha})^2 - (1+l\hat{\alpha}_0)^2\right]^2};$$

$$T = \sum_{n=-N}^{N} \frac{J_n^4(\delta_{\mathrm{FM}})}{(1+n\hat{\alpha})^4} + \frac{\delta_{\mathrm{AM}}^4}{8} \sum_{n=-N}^{N} \sum_{l=-N}^{N} J_n^4(\delta_{\mathrm{FM}}) J_l^4(\delta_{\mathrm{PM}}) \times$$
$$\times \frac{(1+n\hat{\alpha})^4 + 6(1+n\hat{\alpha})^2(1+l\hat{\alpha}_0)^2 + (1+l\hat{\alpha}_0)^4}{\left[(1+n\hat{\alpha})^2 - (1+l\hat{\alpha}_0)^2\right]^4}.$$

From the obtained formulas (7), (8), we will construct graphs of the dependences of the average kinetic energy of a particle on the radiation intensity and the depth of the amplitude, frequency and polarization modulation of an electromagnetic wave to trace the tendency towards a change in the energy characteristics of particles from these parameters (Figure) (graphs created in *Mathcad 15*).



Energy dependences of a charged particle:
a is on the electromagnetic wave intensity for linear polarization (*1*), and circular polarization (*2*); *b* is on different types of modulations of a circularly (*1′–3′*) and linearly (*1–3*) polarized wave (*1* — amplitude; *2* — polarization; *3* — frequency)

The figure shows that the energy of a charged particle depends linearly on the wave intensity and quadratically on the amplitude modulation depth, therefore, radiation modulation contributes more to the energy characteristics of the particle than the intensity, which is one of the reasons for the importance of the problem of achieving high modulation depths, as presented in [23].

Thus, one can resort to increasing the intensity or amplitude modulation depth of an electromagnetic wave to obtain high-energy particles; however, one can control the particle energy by manipulating the frequency and polarization characteristics of the accelerating radiation.

Conclusion. Thus, in this work, a detailed analysis of the motion of a charged particle in the field of a polarization-modulated electromagnetic wave was presented. Expressions for the momentum, velocity, coordinates were obtained and then the obtained expressions were averaged over the oscillation period. Expressions for the average kinetic energy for circular and linear polarization were also obtained. The graphs of the dependences of the average kinetic energy on the radiation intensity and the amplitude modulation depth are presented. The energy properties of a charged particle depend on several parameters of radiation, namely on its intensity, the depth of amplitude modulation, the polarization type, etc. Note that in the absence of amplitude and frequency modulation ($\delta_{\text{AM}} = 0$, $J_n(\delta_{\text{FM}}) = 1$, $\alpha = 0$), formulas (7), (8) are transformed to the form of particle motion in the field of a plane monochromatic wave presented in [15].

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