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# SOLUTION OF A LINEAR NONDEGENERATE MATRIX EQUATION BASED ON THE ZERO DIVISOR

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**Abstract**

New formulas were obtained to solve the linear nondegenerate matrix equations based on zero divisors of numerical matrices. Two theorems were formulated, and a proof to one of them is provided. It is noted that the proof of the second theorem is similar to the proof of the first one. The proved theorem substantiates new formula in solving the equation equivalent in the sense of the solution uniqueness to the known formulas. Its fundamental difference lies in the following: any explicit matrix inversion or determinant calculation is missing; solution is "based" not on the left, but on the right side of the matrix equation; zero divisor method is used (it was never used in classical formulas for solving a matrix equation); zero divisor calculation is reduced to simple operations of permutating the vector elements on the right-hand side of the matrix equation. Examples are provided of applying the proposed method for solving a nondegenerate matrix equation to the numerical matrix equations. High accuracy of the proposed formulas for solving the matrix equations is demonstrated in comparison with standard solvers used in the *MATLAB* environment. Similar problems arise in the synthesis of fast and ultrafast iterative solvers of linear matrix equations, as well as in nonparametric identification of abnormal (emergency) modes in complex technical systems, for example, in the power systems

**Keywords**

*Linear nondegenerate matrix equation, determinant calculation, zero divisor, solution formula*

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**Introduction.** Using such mathematical objects as the matrix zero divisors in theoretical and practical research, analysis and synthesis of control systems for complex technical systems led to significant progress in solving the important applied problems. It should be noted that studies of possibilities to use the matrix zero divisors in system theory are still far from completeness and are constantly in the field of view of the authors. This paper considers the classical problem of finding a solution to a linear nondegenerate matrix equation obtained using the zero-divisor technique.

**Research problem statement.** Let us consider the following linear nondegenerate matrix algebraic equation:

$$\boldsymbol{A}\mathbf{x} = \mathbf{b}, \tag{1}$$

where $\boldsymbol{A} \in \mathbb{R}^{n \times n}$; det $\boldsymbol{A} \neq 0$; $\mathbf{b} \in \mathbb{R}^n / 0$, i.e., the $\mathbf{A}$ real matrix is nondegenerate, $\mathbf{b} \in \mathbb{R}^n$ is a nonzero real vector. This equation is the subject of a large number of works of algebraic and computational nature (see, for example, [1–9]). The most frequently cited result in relation to (1) is the following classical formula:

$$\mathbf{x} = \boldsymbol{A}^{-1}\mathbf{b} = \frac{1}{\det \boldsymbol{A}} \boldsymbol{A}^{\mathrm{V}}\mathbf{b}. \tag{2}$$

Here $\boldsymbol{A}^{\mathrm{V}}$ is the adjoined (allied) matrix.

Another well-known result is called the Cramer formula (rule). Let the matrix have the following form

$$\boldsymbol{A} = \left(\begin{array}{c|c|c|c|c|c} a_{11} & a_{12} & \cdots & a_{1k} & \cdots & a_{1n} \\ \hline a_{21} & a_{22} & \cdots & a_{2k} & \cdots & a_{2n} \\ \hline \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \hline a_{n1} & a_{n2} & \cdots & a_{nk} & \cdots & a_{nn} \end{array}\right), \tag{3}$$

its determinant is

$$\Delta = \det \boldsymbol{A} = \det \left(\begin{array}{c|c|c|c|c|c} a_{11} & a_{12} & \cdots & a_{1k} & \cdots & a_{1n} \\ \hline a_{21} & a_{22} & \cdots & a_{2k} & \cdots & a_{2n} \\ \hline \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \hline a_{n1} & a_{n2} & \cdots & a_{nk} & \cdots & a_{nn} \end{array}\right), \tag{4}$$

then, according to the Cramer rule, solution to the equation (1) has the following form:

$$x_1 = \frac{\Delta_1}{\Delta}, \;\; x_2 = \frac{\Delta_2}{\Delta}, \ldots, \; x_k = \frac{\Delta_k}{\Delta}, \ldots, \; x_n = \frac{\Delta_n}{\Delta}, \tag{5}$$

where

$$\Delta_k = \det \left(\begin{array}{c|c|c|c|c|c} a_{11} & a_{12} & \cdots & b_1 & \cdots & a_{1n} \\ \hline a_{21} & a_{22} & \cdots & b_2 & \cdots & a_{2n} \\ \hline \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \hline a_{n1} & a_{n2} & \cdots & b_n & \cdots & a_{nn} \end{array}\right). \tag{6}$$

Formulas (2) and (4)–(6) have generalizations to the matrix algebraic equation of the following form:

$$\boldsymbol{AX}=\boldsymbol{B},\ \ \boldsymbol{A}\in\mathbb{R}^{n\times n},\ \ \det\boldsymbol{A}\neq 0,\ \ \boldsymbol{B}\in\mathbb{R}^{n\times r}, \tag{7}$$

where in the right-hand side, instead of the $\mathbf{b}\in\mathbb{R}^{n}$ nonzero vector appears nonzero matrix $\boldsymbol{B}\in\mathbb{R}^{n\times r}$.

Existence or presence should be noted of a large number of the iterative solver software packages designed to solve equations (1), (7), where a promising direction of research is using the linear matrix synthesis methods and consideration thereof from the standpoint of control theory in order to create a "solver" capable of operating for a common set of matrices with high velocity and accuracy [10–12]. This applies both too fast and too ultrafast iterative solvers of nondegenerate linear matrix equations (1), (7) [13, 14], and to nonparametric identifiers of abnormal (emergency) modes in complex technical systems, such as the power systems [15]. It is also possible to single out an area built on the basis of original method in multi-step control decomposition and synthesis of a linear multidimensional dynamic system [9, 16] providing rapidly converging transient processes in numerical calculations.

This paper presents *new* formulas for solving equations (1), (7) based on using the zero-divisor method [9].

**Theoretical results and examples of application thereof.** Let us formulate the first statement.

**Theorem 1.** *Solution of the matrix algebraic equation*

$$\boldsymbol{A}\mathbf{x}=\mathbf{b},\ \boldsymbol{A}\in\mathbb{R}^{n\times n},\ \det\boldsymbol{A}\neq 0,\ \ \mathbf{b}\in\mathbb{R}^{n},$$

*is defined by the following formula:*

$$\mathbf{x}=\frac{\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}}{\mathbf{b}^{+}\boldsymbol{A}\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}},$$

*where* $\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}$, $\mathbf{b}^{+}$ *are any vectors satisfying the following equations:*

$$\begin{pmatrix}\mathbf{b}_L^{\perp}\\ \hdashline \mathbf{b}^{+}\end{pmatrix}\mathbf{b}=\begin{pmatrix}\mathbf{0}_{(n-1)}\\ \hdashline 1\end{pmatrix},$$

$$\mathbf{b}_L^{\perp}\boldsymbol{A}\left(\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}\ \middle|\ \left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)^{+}\right)=\left(\mathbf{0}_{(n-1)\times 1}\ \middle|\ \boldsymbol{I}_{(n-1)}\right).$$

Here $\boldsymbol{X}_L^{\perp}$ denotes the left zero divisor of the $X$ maximum rank matrix, $\boldsymbol{X}_R^{\perp}$ is the right zero divisor of the $\boldsymbol{X}$ maximum rank matrix, and $\boldsymbol{X}^{+}$ is the semi-inverse matrix satisfying the Neumann conditions [6], for example, the pseudoinverse Moore — Penrose matrix. To calculate the corresponding zero divisor, algorithms and software products could be used implementing them, such as *SVD, LU* and *QR* decompositions [1, 3]. Canonical Gaussian type decomposition algorithms are used to obtain zero divisors, and examples of their practical application are described in detail in [9].

Let us note that the sets of left and right zero divisors are formed by nondegenerate transformations, respectively, of the rows of left zero divisors and of the columns of right zero divisors [9]. In other words, if at least one zero divisor is defined, then all remaining zero divisors of this rank are formed by appropriate multiplication by the nondegenerate matrix.

The zero-divisor rank maximality condition is uniquely determined by completeness (reversibility) of the composite matrix that includes the original matrix and the zero divisor [9], i.e., zero divisor is a linear transformation, which kernel or cokernel contains the original matrix.

Thus, the $\mathbf{b}_L^{\perp} \in \mathbb{R}^{(n-1)\times n}$ symbol in the Theorem 1 formulation denotes the left zero divisor of the $\mathbf{b} \in \mathbb{R}^n$ vector with the maximum rank $n-1$, i.e., $\mathbf{b}_L^{\perp}\mathbf{b} = 0 \in \mathbb{R}^{(n-1)}$, $\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}$ is the right maximum rank zero divisor of matrix product $\mathbf{b}_L^{\perp}\boldsymbol{A}$, $\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)^{+}$ is the Moore — Penrose pseudoinverse of the $\mathbf{b}_L^{\perp}\boldsymbol{A}$ matrix product.

**Proof of Theorem 1.** Since $\mathbf{b} \in \mathbb{R}^n$, the (non-unique) invertible matrix could be determined:

$$\begin{pmatrix} \mathbf{b}_L^{\perp} \\ \hdashline \mathbf{b}^{+} \end{pmatrix}, \ \mathbf{b}_L^{\perp} \in \mathbb{R}^{(n-1)\times n}, \ \mathbf{b}^{+} \in \mathbb{R}^{1\times n}, \tag{8}$$

satisfying the identity

$$\begin{pmatrix} \mathbf{b}_L^{\perp} \\ \hdashline \mathbf{b}^{+} \end{pmatrix} \mathbf{b} = \begin{pmatrix} \mathbf{0}_{(n-1)} \\ \hdashline 1 \end{pmatrix}, \tag{9}$$

where $\mathbf{0}_{(n-1)\times n}$ is the zero-matrix sizing $(n-1)\times n$; $\mathbf{b}^{+} = \mathbf{b}^T / (\mathbf{b}^T\mathbf{b})$.

Let us consider the following non-degenerate transformation of equation (1):

$$\begin{pmatrix} \mathbf{b}_L^{\perp} \\ \hdashline \mathbf{b}^{+} \end{pmatrix} \boldsymbol{A}\mathbf{x} = \begin{pmatrix} \mathbf{b}_L^{\perp} \\ \hdashline \mathbf{b}^{+} \end{pmatrix} \mathbf{b}. \tag{10}$$

Taking into account (9), from (10) the following is obtained:

$$\begin{pmatrix} \mathbf{b}_L^{\perp} \boldsymbol{A}\mathbf{x} \\ \hdashline \mathbf{b}^{+} \boldsymbol{A}\mathbf{x} \end{pmatrix} = \begin{pmatrix} \mathbf{0}_{(n-1)} \\ \hdashline 1 \end{pmatrix}$$

or if unfolded:

$$\begin{aligned} &\mathbf{b}_L^{\perp} \boldsymbol{A}\mathbf{x} = \mathbf{0}_{(n-1)}; \\ &\mathbf{b}^{+} \boldsymbol{A}\mathbf{x} = 1. \end{aligned} \tag{11}$$

From the first equation (11), provided that $\mathbf{b}_L^{\perp} \boldsymbol{A}$ has the rank equal to $n-1$ follows that

$$\mathbf{x} = \left(\mathbf{b}_L^{\perp} \boldsymbol{A}\right)_R^{\perp} \varphi, \tag{12}$$

it is the right divisor of the $\mathbf{b}_L^{\perp} \boldsymbol{A}$ product. In this case, the $\varphi$ unknown quantity could not be anything other than a scalar.

Substituting formula (12) into the second equation (11) leads to the equation:

$$\mathbf{b}^{+} \boldsymbol{A} \left(\mathbf{b}_L^{\perp} \boldsymbol{A}\right)_R^{\perp} \varphi = 1. \tag{13}$$

Here $\mathbf{b}^{+}$ is the row vector, the $\mathbf{b}^{+} \boldsymbol{A}$ product is the row vector, and, if $\varphi$ is a scalar, then the right zero divisor of the $\mathbf{b}_L^{\perp} \boldsymbol{A}$ product is the column vector. In this case, the $\varphi$ scalar is determined properly from (13) by the following formula:

$$\varphi = \frac{1}{\mathbf{b}^{+} \boldsymbol{A} \left(\mathbf{b}_L^{\perp} \boldsymbol{A}\right)_R^{\perp}}.$$

Returning to (13), let us finally write down the formula:

$$\mathbf{x} = \frac{\left(\mathbf{b}_L^{\perp} \boldsymbol{A}\right)_R^{\perp}}{\mathbf{b}^{+} \boldsymbol{A} \left(\mathbf{b}_L^{\perp} \boldsymbol{A}\right)_R^{\perp}}, \tag{14}$$

which is included in the theorem statement. The proof is complete.

Theorem 1 substantiates a new formula for solving equation $\boldsymbol{A}\mathbf{x} = \mathbf{b}$, which is an equivalent in the sense of the solution uniqueness compared to the known formulas. Its fundamental difference is as follows:

1) explicit $\boldsymbol{A}$ matrix inversion or determinant calculation is missing;

2) solution “rests” not on the left, but on the right side of equation $\boldsymbol{A}\mathbf{x} = \mathbf{b}$;

3) zero-divisor technique is introduced (never used in the classical formulas (2) and (5));

4) the $\mathbf{b}_L^{\perp}$ calculation is reduced to simple operations of permutating elements of vector $\mathbf{b}$;

5) the $\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}$ vector is in fact an envelope spanned on the only solution vector $\mathbf{x}$; in other words, by calculating $\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}$, it is possible to find the $\mathbf{x}$ solution with accuracy up to the $\varphi^{-1} = \mathbf{b}^{+}\boldsymbol{A}\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}$ nonzero factor (see (14)).

Note that in discussing results of this work the following was established: values close to zero appearing in the denominator of formula (14) could worsen the solution conditionality. On the one hand, denominator proximity to zero actually inevitably worsens the solution conditionality. On the other hand, the $\varphi^{-1} = \mathbf{b}^{+}\boldsymbol{A}\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}$ value is the solution conditionality indicator and makes it possible to take the necessary measures before using the found solution (14). Let us demonstrate this with an example.

Let the following be given in equation (1):

$$\boldsymbol{A} = \begin{pmatrix} 0 & -1 & 0 & -1 \\ -2 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}.$$

Let us define a matrix for the $\mathbf{b}$ vector (8):

$$\left(\frac{\mathbf{b}_L^{\perp}}{\mathbf{b}^{+}}\right) = \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \end{array}\right). \tag{15}$$

Next, let us find the matrix:

$$\left(\frac{\mathbf{b}_L^{\perp}}{\mathbf{b}^{+}}\right) = \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \end{array}\right) \tag{16}$$

and the right zero-divisor (16):

$$\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}. \tag{17}$$

Substituting (15)–(17) into (14), the following is obtained

$$\mathbf{x}=\frac{\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}}{\mathbf{b}^{+}\boldsymbol{A}\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}}=$$

$$=\frac{1}{\begin{pmatrix}1&0&0&0\end{pmatrix}\begin{pmatrix}0&-1&0&-1\\-2&1&0&2\\0&1&0&0\\0&0&1&0\end{pmatrix}\begin{pmatrix}1\\0\\1\\1\end{pmatrix}}\begin{pmatrix}1\\0\\1\\1\end{pmatrix}=-1\begin{pmatrix}1\\0\\1\\1\end{pmatrix}=\begin{pmatrix}-1\\0\\-1\\-1\end{pmatrix}.$$

This is the exact solution to the above equation. In the considered example, the scalar $\varphi=-1$.

It is also possible to make sure that by specifying any other suitable $\mathbf{b}_L^{\perp}\boldsymbol{A}$ and $\mathbf{b}^{+}$, for example,

$$\left(\mathbf{b}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}=\begin{pmatrix}\gamma\\0\\\gamma\\\gamma\end{pmatrix},\ \ \gamma\neq 0,\ \ \mathbf{b}^{+}=\begin{pmatrix}\delta+1&\alpha&\beta&\delta\end{pmatrix},$$

the same solution of the equation is determined, and $\varphi=-1/\gamma$. It follows from here that selecting small $\gamma$, when finding the right zero divisor of the $\mathbf{b}_L^{\perp}\boldsymbol{A}$ matrix, provides satisfactory conditionality of the obtained solution.

Theorem 1 is generalized to the case, when on the right-hand side, instead of the $\mathbf{b}\in\mathbb{R}^n$ vector, there appears nonzero matrix $\boldsymbol{B}\in\mathbb{R}^{n\times r}$.

**Theorem 2.** *Solution to the matrix algebraic equation (7) is determined by the following formula:*

$$\boldsymbol{X}=\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}\left(\boldsymbol{B}^{+}\boldsymbol{A}\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}\right)^{-1}, \tag{18}$$

*where* $\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}$, $\boldsymbol{B}^{+}$ *are any matrices satisfying the following conditions,*

$$\begin{pmatrix}\boldsymbol{B}_L^{\perp}\\\hdashline\boldsymbol{B}^{+}\end{pmatrix}\boldsymbol{B}=\begin{pmatrix}\mathbf{0}_{(n-r)\times r}\\\hdashline\boldsymbol{I}_r\end{pmatrix},$$

$$\boldsymbol{B}_L^{\perp}\boldsymbol{A}\left(\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}\ \vdots\ \left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)^{+}\right)=\left(\mathbf{0}_{(n-1)\times r}\ \vdots\ \boldsymbol{I}_r\right).$$

The proof of Theorem 2 is provided by analogy with the proof of Theorem 1. Thus, the following identity is proved:

$$\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}\left(\boldsymbol{B}^{+}\boldsymbol{A}\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}\right)^{-1}=\boldsymbol{A}^{-1}\boldsymbol{B},$$

which, depending on the type of transformation, is being transformed into equivalent identities:

$$\boldsymbol{A}\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}\left(\boldsymbol{B}^{+}\boldsymbol{A}\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}\right)^{-1}=\boldsymbol{B},$$

$$\boldsymbol{A}\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}=\boldsymbol{B}\boldsymbol{B}^{+}\boldsymbol{A}\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}, \tag{19}$$

$$\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}=\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{B}^{+}\boldsymbol{A}\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}.$$

Formula (19) deserves a separate comment. As follows from this formula, the right divisor $\left(\boldsymbol{B}_L^{\perp}\boldsymbol{A}\right)_R^{\perp}$ is not changing, if it is multiplied on the left by matrix $\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{B}^{+}\boldsymbol{A}$, consisting of "mutually inverse" factors $\boldsymbol{A}^{-1}\boldsymbol{B}$ and $\boldsymbol{B}^{+}\boldsymbol{A}$.

It is not difficult to prove that the $\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{B}^{+}\boldsymbol{A}$ matrix is a *projector* [0] with eigenvalues equal to zero and one. So, if we consider matrix $\boldsymbol{A}^{-1}\mathbf{b}\mathbf{b}^{+}\boldsymbol{A}$, where $\mathbf{b}\in\mathbb{R}^{n}$ is the vector, then by virtue of the obvious condition

$$\text{rank}\left(\boldsymbol{A}^{-1}\mathbf{b}\mathbf{b}^{+}\boldsymbol{A}\right)=1$$

this matrix has one unit value and $n-1$ zero values.

For $\boldsymbol{B}\in\mathbb{R}^{n\times r}$ and $\text{rank}\,\boldsymbol{B}=r$, the $\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{B}^{+}\boldsymbol{A}$ matrix would have the $r$ unit and $n-r$ zero eigenvalues.

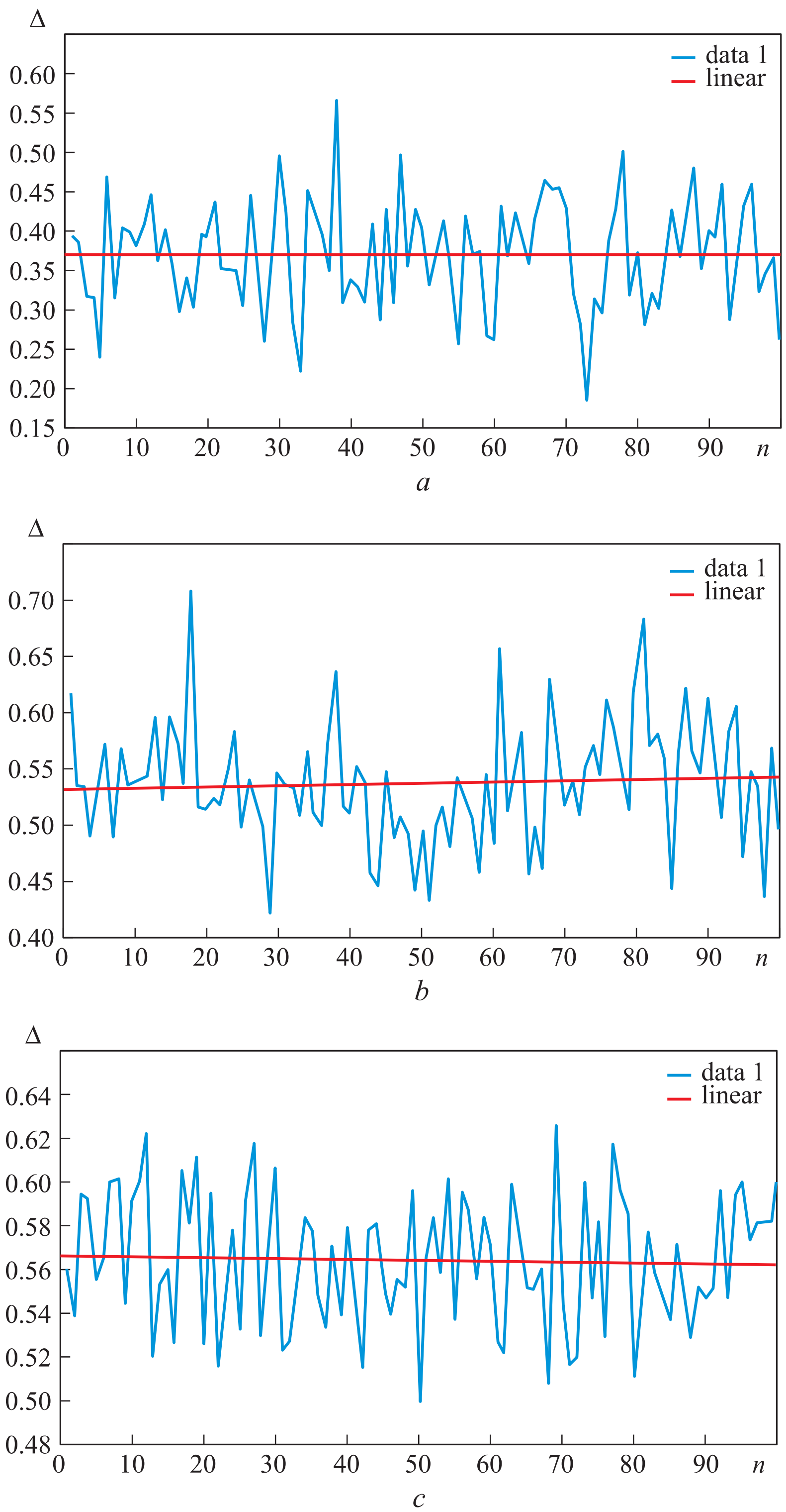
Results of three experiments to evaluate the proposed method accuracy in the *MATLAB* environment are presented in the Figure. In all experiments, 100 matrix equations were solved with $\boldsymbol{A}$ matrices of 300 × 300, 500 × 500, and 700 × 700 sizes. And the $\boldsymbol{B}$ matrices were of 300 × 30, 500 × 30, and 700 × 30 sizes. All matrix elements were set the pseudo-random law using the *randn* operator.

Accuracy was evaluated by analogy with [3] according to the following formula:

$$\Delta=\frac{\left\|\boldsymbol{A}\boldsymbol{X}_{MATLAB}-\boldsymbol{B}\right\|-\left\|\boldsymbol{A}\boldsymbol{X}_{AUT}-\boldsymbol{B}\right\|}{\left\|\boldsymbol{A}\boldsymbol{X}_{MATLAB}-\boldsymbol{B}\right\|}, \tag{20}$$

where $\boldsymbol{X}_{MATLAB}$ is the solution in the *MATLAB* environment found using solver $\mathbf{x}=\boldsymbol{A}\backslash\boldsymbol{B}$; $\boldsymbol{X}_{AUT}$ is the solution found by (18).

In all cases, the proposed method has a significantly higher solution accuracy. It should be mentioned that the accuracy is increasing with enlargement of the matrix sizes (see the mean value behavior, red curve in the Figure).




Function graphs (20) with matrix sizes of 300 × 300 (*a*), 500 × 500 (*b*) and 700 × 700 (*c*)

**Conclusion.** New formulas were obtained for solving the linear nondegenerate matrix equations based on using the matrix zero-divisors technique. High accuracy of the proposed formulas for solving the large dimension matrix equations was demonstrated in comparison with standard solvers used in the *MATLAB* environment. It should be noted that similar problems arise in synthesizing fast and ultrafast iterative solvers of linear matrix equations, as well as in nonparametric identification of the abnormal (emergency) modes in complex technical systems, for example, in the power systems.

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