DIAGONAL-FLEXIBLE SPACES AND ROTOIDS

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This article deals with several non-standard generalizations of the classical concept of a topological group. An important common feature of these generalizations is the fact that all of them are given in geometric terms. They are based on the concept of a diagonal-flexible space. These spaces were introduced and studied in [6] under the slightly different name of a diagonal resolvable space. We provide a brief survey of results obtained so far in this direction, and also obtain some new results on the structure of rotoids which are very close to diagonal-flexible spaces.
One of the main new results below is Theorem 3.1: Every compact rotoid of countable tightness is metrizable. Using this fact, we establish that if X is a compact hereditarily normal strong rotoid, then X is metrizable (Theorem 3.5). The two theorems just mentioned suggest that compact rotoids strongly resemble, by their topological properties, compact topological groups and, more generally, dyadic compacta. Several open problems on rotoids are mentioned, in particular, the next one: is every compact rotoid a dyadic compactum?

Keywords: diagonal-flexible, rectifiable, dyadic compactum, rotoid, topological group, pseudocharacter, π-character, π-base, homogeneous, tightness, G_δ-diagonal, twister , retract.

1. Introduction. General Topology is a framework inside which some of the fundamental ideas of philosophy, such as the ideas of convergence and continuity, can be precisely described, analyzed and applied.

In this article several generalizations of the classical concept of a topological group are discussed. A *topological group* is a group with a topology such that the multiplication is jointly continuous and the inverse operation is continuous. For an introduction to topological groups, see [9] or [20]. The famous Russian mathematician L.S. Pontryagin was one of the founders of Topological Algebra, the domain of mathematics in which the concept of a topological group plays a central role.

The two-headed concept of a topological group is not easy to visualize. About 30 years ago a transparent geometric idea was introduced and used to characterize a class of geometric objects that are very close to topological groups, see about this [11, 13, 15, 23].

If a topological space Y contains two homeomorphic subspaces A and B, then it is natural to ask whether there exists a homeomorphism h of the whole space Y onto itself which maps A onto B.

For a space X, the diagonal Δ_X is the subspace $\{(x, x) : x \in X\}$ of the square $X \times X$. A space X will be called *diagonal-flexible* if, for every $z \in X$, there exists a homeomorphism h of $X \times X$ onto itself such that $h(\Delta_X) = X \times \{z\}$. Note that diagonal-flexible spaces were called *diagonal resolvable spaces* in [6] where they had been introduced.

A space X is *rectifiable*, if there exist $e \in X$ and a homeomorphism g of $X \times X$ onto itself such that $g(x, x) = (x, e)$ and $g(x, y) = (x, z)$, for any $x, y \in X$ and some $z \in X$. A thorough discussion of this notion was given in [11, 12, 15, 23].

It is not difficult to show that every topological group is diagonal-flexible. However, what is really amazing is that it is quite difficult to present an example of a diagonal-flexible space which is not homeomorphic to a topological group.

In this article, a brief survey of known results on diagonal-flexible and rectifiable spaces is provided, a list of attractive open questions is given, and a sample of new results on rotoids is presented, accompanied by an extensive list of references.

In general, our terminology and notation follow [14]. Under a space we understand a Tychonoff topological space. The *tightness* of a space X is countable if the closure of any subset A of X is the union of the closures of countable subsets of A. A compact space B is said to be *dyadic* if it can be represented as a continuous image of a compact topological group. All metrizable compacta are dyadic [14].

2. Some properties of rectifiable spaces and of diagonal-flexible spaces. The closed unit interval $I = [0, 1]$ is not diagonal-flexible. Indeed, the complement of the diagonal in the square $I \times I$ is disconnected, while the complement to $I \times \{0\}$ is connected. Therefore, the diagonal cannot be mapped onto $I \times \{0\}$ by a homeomorphism of the square onto itself. Note, however, that the diagonal in $I \times I$ can be mapped onto $I \times \{1/2\}$ by a homeomorphism of the square onto itself.

The next simple, but quite important, theorem from [11, 23] shows that the class of diagonal-flexible spaces is very wide.

Theorem 2.1. *Every topological group G is a diagonal-flexible space.*

Proof. Put $h(x, y) = (x, xy^{-1})$, for each $(x, y) \in G \times G$. Clearly, h is a homeomorphism of $G \times G$ onto $G \times G$ mapping the diagonal onto $G \times \{e\}$. Since G is homogeneous, we can move the diagonal by a homeomorphism to $G \times \{z\}$ for any $z \in G$ as well. $\square$

Every topological group is, in fact, a rectifiable space, as the above argument demonstrates.

Theorem 2.1 does not generalize to paratopological groups, since the Sorgenfrey line is a paratopological group but is not rectifiable, as was shown in [15]. The next question is open, and it would be very nice to learn the answer to it:

Problem 2.2. *Is every compact diagonal-flexible space rectifiable?*

We see below that not every rectifiable space is homeomorphic to a topological group (the sphere S^7 witnesses this). However, every rectifiable space is homogeneous [15]. The next question remains open:

Problem 2.3. [6] *Is every diagonal-flexible space homogeneous?*

H. Bennett, D. Burke, and D. Lutzer have obtained a result of principal importance in [10]: they have shown that not every diagonal-flexible space is rectifiable. In fact, they established that the Sorgenfrey line is diagonal-flexible, while it is not rectifiable, since every first-countable rectifiable space is metrizable, by an important theorem of A.S. Gul'ko [15]. At present, we have no characterization of rectifiability or of diagonal-flexibility of a space X in purely topological restrictions imposed on X. This is so even if X is compact. it is also not clear, when a compact subspace of a finite-dimensional Euclidean space is diagonal-flexible , and when it is rectifiable. The question is open even for compact manifolds.

For example, the sphere S^n is homeomorphic to a topological group if and only if $n \in \{0, 1, 3\}$. The sphere S^7 is rectifiable, but is not homeomorphic to any topological group [23]. Thus, not every compact rectifiable space is homeomorphic to some topological group. If $n \notin \{0, 1, 3, 7\}$, then S^n is not rectifiable [23].

Problem 2.4. *Is every compact connected* 1*-dimensional rectifiable space metrizable?*

Problem 2.5 *Is every compact connected* 1*-dimensional diagonal-flexible space metrizable?*

Problem 2.6. *Is every compact connected* 1*-dimensional rectifiable space homeomorphic to* S^1*?*

Problem 2.7. *Is every compact connected* 1*-dimensional diagonal-flexible metrizable space homeomorphic to* S^1*?*

The next two simple facts were observed in [6]: the topological product of any family of diagonal-flexible spaces is a diagonal-flexible space, and every discrete space is diagonal-flexible.

However, the free topological sum of diagonal-flexible spaces needn't be diagonal-flexible. This is so, since the next fact holds [6]:

Proposition 2.8. *If a diagonal-flexible space* X *has an isolated point, then* X *is discrete.*

The last statement can be strongly generalized, in the following way:

Theorem 2.9. *If* X *is a diagonal-flexible space, then, for any* $x, y \in X$ *and any open neighbourhood* Oy *of* y *in* X*, there exists an open neighbourhood* Ox *of* x *in* X *and an open continuous mapping* $f : Ox \times \times Ox \to Oy$ *such that* $f((x, x)) = y$.

Proof. Since X is diagonal-flexible, we can fix a homeomorphism h of $X \times X$ onto $X \times X$ such that $h(\Delta_X) = X \times \{y\}$. Then $h((x, x)) = (a, y)$, for some $a \in X$. Then $X \times Oy$ is an open neighbourhood of (a, y) in $X \times X$. Since h is continuous, we can find an open neighbourhood Ox of x in X such that $h(Ox \times Ox) \subset X \times Oy$. The projection p_2 of $X \times X$ to X given by the formula $p_2(x_1, x_2) = x_2$, for any $(x_1, x_2) \in X \times X$, is an open continuous mapping. Therefore, the composition $g = p_2 h$ of h and p_2

is an open continuous mapping of $X \times X$ to X such that $g((x, x)) = y$ and $g(Ox \times Ox) \subset Oy$. The restriction of g to the open subspace $Ox \times Ox$ of $X \times X$ is, clearly, the mapping f we are looking for.

Suppose that X is a topological space with a topology $\mathcal{T}$. Then the *pointwise cardinality* of X at $x \in X$ is the cardinal number $\min\{|Ox| : x \in Ox \in \mathcal{T}\}$ denoted by $|X, x|$. Similarly, we can define the pointwise versions of other cardinal invariants of topological spaces. In particular, $w(X, x)$ will stand below for the pointwise weight of X at $x \in X$.

Applying Theorem 2.9, we obtain:

Corollary 2.10. *If X is a diagonal-flexible space, then, for any $x, y \in X$, we have: $|X, x| = |X, y|$ and $w(X, x) = w(X, y)$.*

Recall that *the character* of a space X at a point x does not exceed τ (notation: $\chi(x, X) \leq \tau$) if there exists a base $\mathcal{B}_x$ at x such that $|\mathcal{B}_x| \leq \tau$.

Corollary 2.11. [6] *If X is a diagonal-flexible space, then, for any $x, y \in X$, we have: $\chi(x, X) = \chi(y, X)$.*

Let τ be an infinite cardinal number. If X has a π-base $\mathcal{B}$ at e, then we write $\pi\chi(e, X) \leq \tau$ and say that the *π-character of X at e* doesn't exceed τ. In this way the π-character $\pi\chi(e, X)$ of X at e is obviously defined. Clearly, we have:

Corollary 2.12. *If X is a diagonal-flexible space, then, for any $x, y \in X$, we have: $\pi\chi(x, X) = \pi\chi(y, X)$.*

Corollary 2.13. *If X is a compact diagonal-flexible space, and X is zero-dimensional at some point, then X is zero-dimensional at every point.*

Theorem 2.9 and Corollaries 2.10, 2.13 and 2.12 are new results. We will expose below a few advanced results on diagonal-flexible spaces that have been established in [6].

Many results presented below are intimately related to the next question which is still open:

Problem 2.14. [6] *Is every diagonal-flexible compact space dyadic?*

Note in this connection that every rectifiable compact space is a Dugundji compactum [23] and therefore, is a dyadic compactum. See [14] and [9] for the basic properties of dyadic compacta.

Problem 2.15. *Is it true in ZFC that if a sequential compact space F is diagonal-flexible, then F is metrizable?*

In connection with this open problem, we present here some partial results proved in [6]:

Theorem 2.16. [6] *If a diagonal-flexible compact space F is first-countable at some point, then F is metrizable.*

In particular, this theorem is applicable to first-countable compacta and to Corson compacta and Eberlein compacta [3].

Corollary 2.17. [6] *Every diagonal-flexible Corson compactum is metrizable.*

Under (CH), Theorem 2.16 also covers the cases of diagonal-flexible sequential compacta and diagonal-flexible compacta of the cardinality $\leq 2^{\omega}$. Every compact rectifiable space of countable tightness is metrizable [15, 23]. It is not yet known whether a similar statement can be proved in ZFC for diagonal-flexible compacta of countable tightness. However, we have the following result:

Theorem 2.18. [6] *If (CH) holds, then every diagonal-flexible compact sequential space X is metrizable.*

Proof. Under (CH), every nonempty sequential compactum is first-countable at some point [8]. It remains to apply Theorem 2.16.

Consistently, every diagonal-flexible compact space of countable tightness is metrizable [6].

Theorem 2.19. [6] *Is every locally compact diagonal-flexible space paracompact?*

Note that every locally compact topological group is paracompact [9].

A space X has a *regular G_δ-diagonal* if there exists a countable family of open neighbourhoods of the diagonal in $X \times X$ the intersection of the closures of which in $X \times X$ is the diagonal Δ_X [18].

Proposition 2.20. [6] *If a diagonal-flexible space X is first-countable at some point, then X has a regular G_δ-diagonal.*

Theorem 2.21. [6] *If a diagonal-flexible pseudocompact space X has a G_δ-point, then X is metrizable (and compact).*

Proof. Since the space X is pseudocompact, it is first-countable at every G_δ-point. By Proposition 2.20, X has a regular G_δ-diagonal. Since every pseudocompact space with a regular G_δ-diagonal is metrizable [7, 18], the space X is metrizable. $\square$

A powerful general metrization theorem for rectifiable spaces has been obtained by A.S. Gul'ko [15]:

Theorem 2.22. *A nonempty rectifiable space X is metrizable if and only X has a countable π-base at some point $e \in X$.*

This result doesn't extend to all diagonal-flexible spaces, since the Sorgenfrey line is a first-countable non-metrizable diagonal-flexible space [10].

Problem 2.23. *Is every diagonal-flexible space with countable π-character first-countable?*

Problem 2.24. *Is every diagonal-flexible compact space with countable π-character first-countable?*

3. Rotoids. In this section, we present several new results on spaces which are very close to diagonal-flexible spaces. They are called rotoids. In particular, we answer a question raised in [6, Problem 8.23], and discuss some corollaries of this result. We also formulate some open questions.

One of the main results below is Theorem 3.1: every compact rotoid of countable tightness is metrizable. Using this result, we establish that if X is a compact rotoid such that every discrete subspace of X is countable, then X is metrizable (Theorem 3.3). We also show that if X is a compact hereditarily normal strong rotoid, then X is metrizable (Theorem 3.5).

These results demonstrate that the class of compact rotoids introduced in [6] is very close to the class of dyadic compacta: practically, every classical condition for metrizability of a dyadi compactum turns out to guarantee metrizability of any compact rotoid as well. These results make plausible the conjecture in [6] that every compact rotoid is a dyadic compactum (Problem 8.19). However, whether this is so remains unknown. I believe that the answer “yes” to the conjecture would be a fascinating result. Of course, it would be a far reaching generalization of the famous Ivanovskij-Kuz’minov Theorem on dyadicity of compact topological groups (see about this [9]).

Below τ stands for an infinite cardinal number.

A set $A \subset X$ will be called a G_τ-subset of X, if there exists a family γ of open sets in X such that $|\gamma| \leq \tau$ and $A = \cap\gamma$. If $x \in X$ and $\{x\}$ is a G_τ-subset of X, then we say that x is a G_τ-point in X. In this case we also say that the pseudocharacter of X at x does not exceed τ, and write $\psi(x, X) \leq \tau$.

The $\pi\tau$-character of a space X at a point $e \in X$ is not greater than τ (denoted by $\pi\tau\chi(e, X) \leq \tau$) if there exists a family γ of nonempty G_τ-sets in X such that $|\gamma| \leq \tau$ and every open neighbourhood of e contains at least one element of γ. Any such family γ is called *a $\pi\tau$-network* at e. If $\tau = \omega$, we rather use expressions $\pi\omega$-character and $\pi\omega$-network. In particular, if X has a countable π-base at e, then $\pi\omega\chi(e, X) \leq \omega$.

A space X is a rotoid if there exists $e \in X$ and a homeomorphism h of $X \times X$ onto itself such that $h((x, e)) = (x, x)$ and $h((e, x)) = (e, x)$, for every $x \in X$ [6].

If e in this definition can be chosen to be an arbitrary element of X, we say that X is a strong rotoid. Clearly, every homogeneous rotoid is a strong rotoid.

Notice that every rectifiable space is a strong rotoid. Hence, every topological group is a strong rotoid [6].

3.1. Main new results. Here we just formulate the main results. Their proofs are given in the next subsection.

Theorem 3.1. *Every compact rotoid of countable tightness is metrizable.*

Theorem 3.2. *If X is a countably compact rotoid such that the π-character of X is countable at every point of X, then X is metrizable.*

In connection with the last two statements, we notice that there exists a non-metrizable countably compact strong rotoid X with countable tightness: to see this, it is enough to take the standard Σ-product of uncountably many copies of the discrete group D consisting of just two elements. In particular, we see that Theorem 3.1 cannot be extended to countably compact rotoids.

Theorem 3.3. *If X is a compact rotoid such that every discrete subspace of X is countable, then X is metrizable.*

Theorem 3.4. *If X is a compact strong rotoid such that the π-character of X at some point of X is countable, then X is metrizable.*

Recall that a space is *hereditarily normal* if every subspace of it is normal.

Theorem 3.5. *If X is a compact hereditarily normal strong rotoid, then X is metrizable.*

Theorem 3.6. *If Z is a retract of a strong rotoid X, and Z is a compactum with countable tightness, then Z is first-countable.*

Theorem 3.7. *If Z is a retract of a strong rotoid X, and Z is compact, then Z is first-countable at every point z at which Z has a countable π-base.*

3.2. Proofs of the main results. B.E. Shapirovskij has shown [21] that if X is any compact space of countable tightness, then the π-character of X at every point of X is countable. Hence, Theorem 3.1 immediately follows from Theorem 3.2.

If every discrete subspace of a compact space X is countable, then the tightness of X is countable as well, by a result in [1]. Therefore, Theorem 3.3 follows from Theorem 3.1. Thus, to prove the first three theorems, it is enough to prove Theorem 3.2.

To do this, we recall some techniques from [4].

A twister at a point e of a space X is a binary operation on X, written as xy for x, y in X, satisfying the following conditions:

a) $ex = xe = x$, for each $x \in X$;

b) for every $y \in X$ and every open neighbourhood V of y, there is an open neighbourhood W of e such that $Wy \subset V$; and

c) if $e \in \overline{B}$, for some $B \subset X$, then, for every $x \in X$, $x \in \overline{xB}$.

If a space X has a twister at $e \in X$, then we say that X is *twistable at* e. A space is *twistable* if it is twistable at every point. Twistability was introduced and applied in [4], [5]. The next fact has been established in [4]. For the sake of completeness, we provide its proof.

Lemma 3.8. *Suppose that X is a space twistable at a point $e \in X$, and that $\pi\tau\chi(e, X) \leq \tau$, for some cardinal number τ. Then $\psi(e, X) \leq \tau$.*

Proof. Fix a twister at e, and let γ be a $\pi\tau$-network at e. Take any $V \in \gamma$, and fix $y_V \in V$.

There exists a G_τ-set P_V in X such that $e \in P_V$ and $P_V y_V \subset V$. Put $Q = \cap\{P_V : V \in \gamma\}$. Clearly, Q is a G_τ-set and $e \in Q$.

Claim: $Q = \{e\}$. Assume the contrary. Then we can fix $x \in Q$ such that $x \neq e$. There exist open sets U and W such that $x \in U$, $e \in W$, and $U \cap W = \varnothing$. Since $xe = x \in U$, and the multiplication on the left is continuous at e, we can also assume that $xW \subset U$.

Since γ is a $\pi\tau$-network at e, there exists $V \in \gamma$ such that $V \subset W$. Then for the point y_V we have: $y_V \in W$, $xy_V \in P_V y_V \subset V \subset W$, and $xy_V \in xV \subset xW \subset U$. Hence, $xy_V \in W \cap U$ and $W \cap U \neq \varnothing$, a contradiction. It follows that $Q = \{e\}$. Thus, $\psi(e, X) \leq \tau$. □

To prove Theorem 3.2, we need one more technical result:

Propositional 3.9. *Every rotoid X is twistable at some point. Every strong rotoid is twistable at every point.*

Proof. Since X is a rotoid, we can fix $e \in X$ and a homeomorphism h of $X \times X$ onto itself such that $h((x, e)) = (x, x)$ and $h((e, x)) = (e, x)$, for every $x \in X$. Now we define a binary operation on X by the rule: for $x, y \in X$, put $xy = z$, where z is the second coordinate of $h((x, y))$. Since h is continuous, this binary operation is continuous. Obviously, $ex = x = xe$, for any $x \in X$. Hence, the binary operation so defined is a twister. The second statement is proved similarly. □

The proof of Proposition 5.1 from [6] given there proves a somewhat stronger statement. This argument concerns a space X with a G_δ-point a and a homeomorphism g of $X \times X$ onto itself such that $g(\Delta_X) = X \times \{e\}$, for some $e \in X$. It is not assumed that e coincides with a. The argument in [6] proceeds as follows:

"$\ldots g(a, a) = (c, e)$, for some $c \in X$, and (a, a) is a G_δ-point in $X \times X$. It follows that (c, e) is also a G_δ-point in $X \times X$. Therefore, e is a G_δ-point in X which implies that the set $X^e = X \times \{e\}$ is a G_δ-set in $X \times X$. Since g is a homeomorphism, it follows that the diagonal Δ_X is a G_δ-set in $X \times X$."

Thus, the argument above proves that the next statement holds:

Propositional 3.10. *If X is a space with a G_δ-point a, and g is a homeomorphism of $X \times X$ onto itself such that $g(\Delta_X) = X \times \{e\}$, for some $e \in X$, then the diagonal Δ_X is a G_δ-set in $X \times X$.*

Corollary 3.11. *If X is a rotoid such that at least one point of X is a G_δ-point in X, then the diagonal Δ_X is a G_δ-set in $X \times X$.*

Corollary 3.11 is more general than Proposition 5.1 in [6]. It is also more general than Proposition 4.6 in [10].

Proof of Theorem 3.2. By Proposition 3.9, X is twistable at some $e \in X$. Since the π-character of X at e is countable, it follows from Lemma 3.8 that e is a G_δ-point in X. Therefore, by Corollary 3.11, X has a G_δ-diagonal in $X \times X$. Hence, X is metrizable, by J. Chaber's Theorem, since X is countably compact and [14] □

The proof of Theorem 3.4 practically coincides with the proof of Theorem 3.2, only a few obvious minor changes are needed.

Proof of Theorem 3.5. The space X cannot be continuously mapped onto the Tychonoff cube I^{ω_1}, since the space I^{ω_1} is not hereditarily normal. Hence, by a theorem of Shapirovskij in [22], the π-character of X at some point $e \in X$ is countable. Since X is a strong rotoid, the space X is twistable at e. Now it follows from Lemma 3.8 that e is a G_δ-point in X. Therefore, X has a G_δ-diagonal in $X \times X$ by Corollary 3.11. Hence, X is metrizable, since X is compact. $\square$

To prove Theorem 3.7, we need the following elementary fact from [4]:

Propositional 3.12. *If Z is a retract of X, and $e \in Z$, and there is a twister at e on X, then there is a twister on Z at e.*

Theorem 3.7 immediately follows from the next statement:

Theorem 3.13. *If Z is a retract of a strong rotoid X, and Z is a space of point-countable type, then Z is first-countable at every point z at which the space Z has a countable π-base.*

Proof. Since X is a strong rotoid, Propositions 3.12 and 3.9 imply that Z is twistable at every point.

Take any point z at which the space Z has a countable π-base. Now Lemma 3.8 implies that z is a G_δ-point in Z. Since Z is of point-countable type, it follows that the space Z is first-countable at z. $\square$

Proof of Theorem 3.6. By a theorem of Shapirovskij in [21], the π-character of Z at every point of Z is countable, since it doesn't exceed the tightness of Z. Now Theorem 3.6 follows from Theorem 3.7 (or from Theorem 3.13). $\square$

3.3. Some open questions. Some of the above results easily generalize to certain larger classes of spaces. For example, Theorems 3.1–3.5 remain true for paracompact p-spaces, with practically the same proofs.

However, some interesting questions on compact rotoids remain open. We refer the reader to [6], where many questions on rotoids and close to them spaces have been formulated. The majority of these questions remain open.

Problem 3.14. [6] *Is every compact rotoid a dyadic compactum?*

Problem 3.15. [6] *Is every compact rotoid a Dugundji compactum?*

Example 3.16. It was shown in [10] (Proposition 4.1) that the closed unit interval $I = [0, 1]$ is a rotoid. However, the space $[0, 1]$ is not rectifiable, since it is not homogeneous. Therefore, not every compact rotoid is rectifiable. We will now strengthen this conclusion. Put $X = I^\omega$. Clearly, X is also a rotoid. But X is also homogeneous. Therefore, X is a strong rotoid. However, X is not rectifiable, as it was shown in [15].

The next question is closely related to Problems 3.14 and 3.15.

Problem 3.17. *Is the Souslin number of any compact (strong) rotoid countable?*

Recall that a space X is *Mal'tsev* if there exists a continuous mapping $F : X^3 \to X$ such that $F(x, x, y) = y = F(y, x, x)$, for every $x, y \in X$. The Souslin number of every Mal'tsev compactum is countable (see [23], [9]). Thus, Problem 3.18 below is related to Problem 3.17.

Problem 3.18. *Is every compact rotoid a Mal'tsev space?*

Problem 3.19. *Is every compact strong rotoid homogeneous?*

In the non-compact case, rotoids and strong rotoids are not so close to topological groups as in the compact case. This became clear after the following remarkable result in [10]: *Sorgenfrey line is a strong rotoid*. Thus, a first-countable strong rotoid needn't be metrizable, unlike a topological group.

Observe that Sorgenfrey line is a paratopological group. The answer to the next question, motivated by the result in [10] we just mentioned, will be probably in the negative:

Problem 3.20. *Is every paratopological group a rotoid?*

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