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A NEW LOOK AT FUNDAMENTALS OF THE PHOTOMETRIC LIGHT TRANSPORT AND SCATTERING THEORY. PART 2: ONE-DIMENSIONAL SCATTERING WITH ABSORPTION

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Abstract

In the first part of the article, one-dimensional (1D) pure scattering processes were taken into detailed consideration. It allowed to prove that the scattering coefficient is not just a real optical property of a turbid medium, but also is a parameter of the mathematical description of the problem. It depends on the approximation, which is applied to solve the problem. Therefore, in different approaches it can vary. More real and close to realistic practical problems are scattering problems with absorption. This second part of the article describes the 1D scattering problems with absorption. It is shown, that scattering and absorption processes inside the light-scattering medium are not independent in most cases, so a formulation of the first coefficients of initial differential equations, which mathematically describe the problem, as the simplest superposition of scattering and absorption coefficients is wrong. Inaccuracy in this formulations leads to inaccuracies in final results. More correct formulation, for example, in application to the classical two-flux Kubelka — Munk (KM) approach, which is a good 1D limit for the radiative transport equation, allows one to obtain the exact analytical solution for boundary radiant fluxes (backscattered and transmitted ones), contrary to the classic KM approximation. In addition, it leads to the need for revision of definitions of a number of basic terms in the general radiative transport theory, especially of the albedo, which plays a key role in Monte-Carlo simulations

Keywords

Scattering, absorption, light transport, radiative transport equations, Kubelka — Munk approach, single scattering approximation, multiple scattering

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Introduction. As previously, we will consider only stationary, time-independent problems, because they are exactly the fundamentals of the phenomenological light transport and scattering theory (LT&ST). Also, we will use the same 1D model of the scattering medium with a number of discrete optical heterogeneities inside the medium. The study described in this second part of the article was aimed at finding answers to the problem, how we can use our previously obtained results on the scattering coefficient definition in different cases of scattering with absorption. In most cases, the interaction of light with turbid media can mainly be characterized by two phenomena: absorption and scattering. However, the distinction between absorption and scattering is not always clearly understood from 1D differential transport equations, which mathematically describe the problem. It is by far the most important problem, in our opinion. In elastic scattering no energy loss occurs in the scattering process, and scatterers are fundamentally distributed discretely inside a light-scattering medium. At the same time, the absorption reduces the flux energy, but the type of a distribution of absorbers inside the medium (continuous or discrete) between scatterers, as it was shown in the introduction to the first part of the article [1], does not matter. Are the definitions of the scattering coefficient previously obtained applicable in this complex situation?

Scattering 1D problem with absorption. Now we are ready to complicate our approach and to solve the scattering problem when absorption of radiation in each interval between heterogeneities inside the medium exists. For a simplicity, let us assume that all intervals between heterogeneities are of the same length h (thickness), i. e., they are spread uniformly inside the medium, and all of the intervals (substances in the interval) have the same identical coefficient of absorption — μ_a. Also, let the first and the last heterogeneities are located at the distance of $h/2$ from the external borders of the medium to have together the length h. Scattering in the medium is simulated as previously by infinitely thin reflecting borders of heterogeneities r_1, r_2, ..., r_n. Figure 1 illustrates the model.

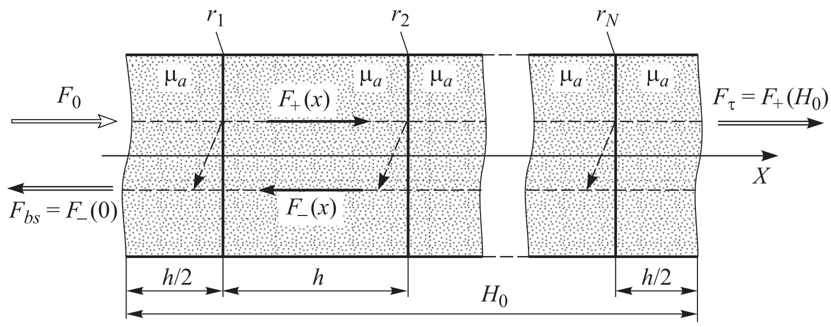

Fig. 1. Model of 1D scattering media with absorption

This model is a good approximation of biological tissue with sufficiently large heterogeneities. Such models have long been known in physics and optics as pile models by Stokes [2]. Benford [3, 4] also published a detailed analysis of the absorption and scattering using assumption that the sample was divided into a series of plane parallel layers. In contrast to the Stokes problem, which considers a pile of thick plates, infinitely thin reflecting heterogeneities, which are included in a one thick plate, are considered in our model. In addition, in contrast to the Stokes problem, reflection from external boundary of the medium is neglected, because the borders are considered rough (friable) [5]. Rough external borders correspond well to coarse surface of biological tissue or to a surface of powdered materials. Reflection from such surfaces is negligible as compared to backscattered radiation. Indeed, for the 1D model, the term "rough external border" is arbitrary. Most of all, it is suitable for 2D and (or) 3D problems. Nevertheless, here it is used for the sake of physical illustration, to highlight and to "justify" the lack of reflection at external borders. We have also to note, that the uniform distribution of all heterogeneities inside the medium does not reduce the theoretical generality of the problem. It can be shown that at any random distribution of the thicknesses of intervals h inside the medium, the statistically averaged outputs for backscattered and transmitted fluxes will be the same [6].

Single scattering approximation. Once again, we would like to start with the single scattering approximation (SSA). At SSA, the forward radiant flux $F_+(x)$ is scattered and absorbed along its path, but the backward flux $F_-(x)$ being formed can be only absorbed. Classic two-flux approach dictates for this scheme the coupled system of the linear differential equations as follows:

$$\frac{dF_+(x)}{dx} = -\beta_1 F_+(x);$$
$$\frac{F_-(x)}{dx} = KF_-(x) - \beta_2 F_+(x), \tag{1}$$

where β_1 is the extinction coefficient due to the scattering and absorption, β_2 is the unknown yet scattering (backscattering) coefficient that forms the backward flux $F_-(x)$, and $K = \mu_a$ is the absorption coefficient. Here, for the backward flux $F_-(x)$ we can directly accept the equation $K = \mu_a$, since the absorption occurs between inhomogeneities, and the secondary scattering is absent, i. e., for the $F_-(x)$ our 1D medium at SSA is not a turbid medium, just an absorptive one like it was considered in the introduction.

Usually, in the classic radiative transport equation (RTE) by default it is *assumed* that:

$$\beta_1 = K + S \quad \text{and} \quad \beta_2 = S, \tag{2}$$

This linear superposition for β_1 is the common *a-priory* heuristic assumption in LT&ST. Could be it proved rigorously? Direct calculation of the decrement of the forward flux inside Δx yields:

$$\Delta F_+ = F_+(x+\Delta x) - F_+(x) = F_+(x)(1-R)^N e^{-\mu_a \Delta x} -$$
$$-F_+(x) = F_+(x)\left[(1-R)^{\mu_\rho \Delta x} e^{-\mu_a \Delta x} - 1\right]. \tag{3}$$

Therefore, the differential of the forward flux is:

$$\frac{dF_+(x)}{dx} = \lim_{\Delta x \to 0} \frac{\Delta F_+}{\Delta x} = F_+(x)\left[\mu_\rho \ln(1-R) - \mu_a\right] = -(K+S)F_+(x), \tag{4}$$

where we see the appearance of the scattering coefficient S in the form of Eq. (22), part 1 [1], for SSA. Nothing has changed for it yet. Moreover, the linear superposition Eq. (2) for β_1 can be considered as proven at the presence of absorption for SSA. It yields the known solution:

$$F_+(x) = F_0 e^{-(K+S)x}, \tag{5}$$

where $K = \mu_a$ and $S = -\mu_\rho \ln(1-R)$. The function remains exponential.

Classic result for $F_-(x)$ can be derived from the second equation of the system (1). With the use of Eq. (5), the second equation is the inhomogeneous first-order linear constant coefficient of ordinary differential equation of the form:

$$\frac{df(x)}{dx} - Kf(x) = g(x), \tag{6}$$

solution of which can be obtained as a product of two functions $f(x) = u(x)v(x)$ at $v(x) = e^{Kx}$ and $u(x) = \int \frac{g(x)}{v(x)} dx + C.$ Constant C can be determined from the boundary condition $F_-(H_0) = 0$. Therefore:

$$F_-(x) = \frac{F_0 \beta_2 e^{Kx}}{2K+S}\left[e^{-(2K+S)x} - e^{-(2K+S)H_0}\right]. \tag{7}$$

Using Eq. (7), one can write the backscattered flux F_{BS} as follows:

$$F_{BS} = F_-(0) = \frac{\beta_2 F_0}{2K+S}\left(1 - e^{-(2K+S)H_0}\right). \tag{8}$$

Now we need to determine β_2 through real physical properties of the medium — μ_a, μ_ρ, R, H_0. Consider the direct calculation scheme for the increment of $F_-(x)$ inside Δx as shown in Fig. 2.

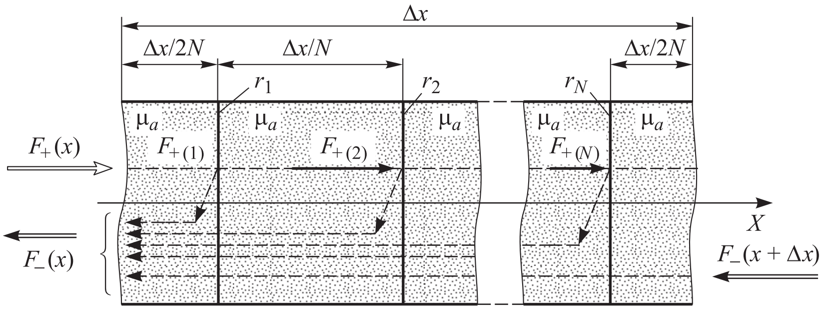

Fig. 2. Formation of the increment of $F_-(x)$ inside Δx

Note, that in this way $F_{+(i)} = F_+(x) e^{-\mu_a \frac{\Delta x}{2N}} (1-R)^{i-1} e^{-\mu_a \frac{\Delta x(i-1)}{N}}$. It forms the series of $F_{-(i)}$ as follows: $F_{-(1)} = F_+(x) e^{-\mu_a \frac{\Delta x}{N}} R$; $F_{+(2)} = F_+(x) e^{-\mu_a \frac{3\Delta x}{N}} R(1-R)$; $F_{+(3)} = F_+(x) e^{-\mu_a \frac{5\Delta x}{N}} R(1-R)^2$; … etc., which is the usual decreasing geometrical progression. In addition, $F_{-}(x)$ is formed by the term $F_{-}(x+\Delta x)e^{-\mu_a \Delta x}$. So, the total $F_{-}(x)$ is determined by the sum:

$$F_{-}(x) = F_{+}(x) e^{-\mu_a \frac{\Delta x}{N}} R \frac{1-(1-R)^N e^{-2\mu_a \Delta x}}{1-(1-R) e^{-2\mu_a \frac{\Delta x}{N}}} + F_{-}(x+\Delta x) e^{-\mu_a \Delta x}. \qquad (9)$$

Here $N = \mu_\rho \Delta x$ is the total number of heterogeneities inside Δx. Therefore:

$$\frac{dF_{-}(x)}{dx} = \lim_{\Delta x \to 0} \left(\frac{F_{-}(x+\Delta x) - F_{-}(x)}{\Delta x} \right) = \mu_a F_{-}(x) - \left[\frac{R(S+2\mu_a) e^{-\mu_a/\mu_\rho}}{1-(1-R) e^{-2\mu_a/\mu_\rho}} \right] F_{+}(x). \qquad (10)$$

Comparing Eq. (10) and the second equation of the system (1), we found out that:

$$\beta_2 = \frac{R(S+2\mu_a) e^{-\mu_a/\mu_\rho}}{1-(1-R) e^{-2\mu_a/\mu_\rho}}. \qquad (11)$$

Thus, the second assumption in Eqs. (2) is wrong! It becomes true as an extreme case only if $\mu_a \to 0$, so the second expression in Eqs. (2) is the *particular case* of the Eq. (11). Once again, we see that the "scattering coefficient", in this instance it is β_2, gives for us unexpected result and takes a new form, depending on the mathematical formulation of the problem. Moreover, in this example β_2, which forms and enhances $F_{-}(x)$, is not equal to S, which reduces $F_{+}(x)$. The ratio β_2 / S as a function of μ_a / μ_ρ is shown in Fig. 3. One can see, that always $\beta_2 < S$. It means that the radiation transformed into the $F_{-}(x)$ is smaller than the radiation backscattered from $F_{+}(x)$. Part of the scattered radiation is absorbed directly inside Δx. What is also important, comparing Eqs. (7) and (5) one can found out the difference in the exponential attenuation. Forward flux $F_{+}(x)$ is reduced faster.

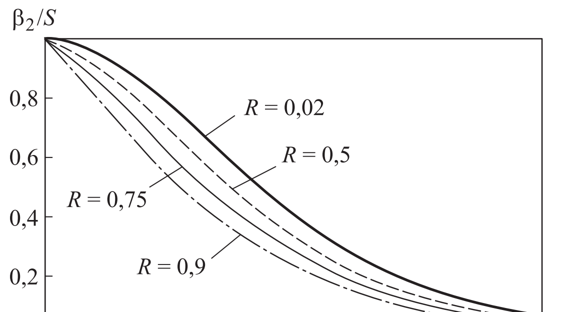

Fig. 3. The ratio β_2 / S as a function of the parameter μ_a / μ_ρ

Multiple scattering approach. Indeed, the most interesting case is the case of multiple scattering. The system of ordinary linear differential equations describing the multiple scattering at the presence of absorption is the system of the coupled equations:

$$\frac{dF_+(x)}{dx} = -\beta_1 F_+(x) + \beta_2 F_-(x);$$
$$\frac{F_-(x)}{dx} = \beta_1 F_-(x) - \beta_2 F_+(x). \quad (12)$$

Under assumptions

$$\beta_1 = (K + S) \quad \text{and} \quad \beta_2 = S, \quad (13)$$

this system is well-known as the Kubelka — Munk (KM) system [7, 8]. However, it is obvious now, that we should be careful with these assumptions (13). We have seen, that the first assumption is valid at single scattering if to define

$$S = -\mu_\rho \ln(1 - R), \quad (14)$$

as it was obtained at SSA in the first part of the article (see Eq. (22) [1]), but the second one is also valid for the situation, when absorption is very small, close to zero. Once absorption becomes significant, the Eq. (11) should be used for β_2 at SSA.

In the general case for $\beta_1 \neq \beta_2$ (the case when $\beta_1 = \beta_2$ is identical to Eqs. (23), part 1 [1]), the solution of the system (12) is known:

$$F_+(x) = C_1 e^{-\alpha x} + C_2 e^{\alpha x}; \; F_-(x) = C_1 A_- e^{-\alpha x} + C_2 A_+ e^{\alpha x}, \quad (15)$$

where C_1 and C_2 are integration constants determined from boundary conditions, $\alpha = \sqrt{\beta_1^2 - \beta_2^2}$, $A_+ = \beta_2 / (\beta_1 - \alpha)$, $A_- = 1/A_+$. This is the standard, well-known, pure mathematical result of the integration of the system (12). To establish the physical meaning of the scattering coefficient, it is necessary to determine β_1 and β_2 through the optical properties of the turbid medium.

As previously, we need to consider two different cases (two subcases) — SSA inside Δx and multiple scattering over the whole medium in a macroscopic sense, and MSA directly inside Δx. The first case, as we have got already in the section *Single scattering approximation*, is described by equations:

$$\beta_1 = \mu_a + S;$$
$$S = -\mu_\rho \ln(1 - R); \quad (16)$$
$$\beta_2 = \frac{R(S + 2\mu_a) e^{-\mu_a/\mu_\rho}}{1 - (1 - R) e^{-2\mu_a/\mu_\rho}}.$$

The case $\beta_2 = -\mu_\rho \ln(1 - R)$ instead of the last equation in (16) can be taken into consideration, as well. However, the Fig. 3 immediately explains for us, that this way will lead just to the enhanced backscattered flux if to compare with Eqs. (16), not more. Merely, in this case some part of the absorbed radiation inside Δx on its conversion way to a backward flux will not be taken into account. We have to highlight here, that we come to a very important result. In the general case, absorption of radiation inside Δx is determined by a subtraction $\Delta\beta = \beta_1 - \beta_2$ [9], which is not always equal to μ_a (!). Since β_2 is less than S, absorption in the case of Eqs. (16) will be determined not only by μ_a, but also by scattering properties of the turbid medium, such as R, μ_ρ, etc.

The case of multiple scattering inside Δx is more complex. In this case (the most general case of 1D problems), rigorous analytical solutions for parameters β_1 and β_2 was derived as follows [10]:

$$\beta_1 = \omega \frac{\mu_a - \mu_\rho \ln(1-R) + \mu_\rho \ln(1-\omega+\sqrt{\omega^2 - R^2 e^{-2\mu_a/\mu_\rho}})}{\sqrt{\omega^2 - R^2 e^{-2\mu_a/\mu_\rho}}}; \tag{17a}$$

$$\beta_2 = R e^{-\mu_a/\mu_\rho} \frac{\mu_a - \mu_\rho \ln(1-R) + \mu_\rho \ln(1-\omega+\sqrt{\omega^2 - R^2 e^{-2\mu_a/\mu_\rho}})}{\sqrt{\omega^2 - R^2 e^{-2\mu_a/\mu_\rho}}}, \tag{17b}$$

where we denoted $\omega = \frac{1-(1-2R)e^{-2\mu_a/\mu_\rho}}{2}$. Once again, one can see, that SSA and MSA inside Δx lead to different results for the scattering coefficient β_2, as well as for the extinction coefficient β_1. Moreover, Eqs. (17a), (17b) turn out to be the most general case of equations among all previously obtained results. For example, for the medium without absorption ($\mu_a = 0$) their extreme values are:

$$\lim_{\mu_a \to 0} \beta_1 = \lim_{\mu_a \to 0} \beta_2 = \beta_m = \frac{\mu_\rho R}{1-R}, \tag{18}$$

that corresponds well with the Eq. (36), part 1 [1]:

$$S = \frac{\mu_\rho R}{1-R}. \tag{19}$$

In the case of $R = 0$ (pure absorption), $\beta_2 = 0$, and $\beta_1 = \mu_a = K$, and so on. Eqs. (17a), (17b) prove, that in general in the LT&ST there are a number of tasks where we cannot separate absorption and scattering coefficients (K and S in the KM notations and μ_a and μ_s in the RTE notations) in the extinction coefficient β_1. Such a separation and assumption of validity of Eqs. (13) is a direct consequence of the accepted phenomenological formalism about independence of absorption and scattering processes. It was not proved previously, just theoretically assumed and accepted. However, now it can be rigorously proved that absorption and scattering processes are not independent. We cannot write a decomposition $\beta_1 = \mu_a + \beta_2$, or $\beta_1 = K + S$, because it follows from (17a), (17b), that:

$$\beta_1 = \frac{\omega e^{\mu_a/\mu_\rho}}{R} \beta_2. \tag{20}$$

It comes into operation the so-called in photometry (and quite forgotten today) *Photometric Invariant* "*J*", introduced by Gurevich [11]:

$$\frac{1+(F_{BS}/F_0)^2-(F_\tau/F_0)^2}{2(F_{BS}/F_0)}=J=\text{const},\tag{21}$$

where J is the Gershun — Gurevich invariant, which is independent on thickness of the medium of light propagation and reflects native optical properties of Δx. It can be easily shown, for example, using the simplest solution of the system (12) together with Eqs. (17a) and (17b) for semi-infinite medium when $F_\tau = 0$, that in our 1D scattering model:

$$J=\frac{\omega e^{\mu_a/\mu_\rho}}{R}=\frac{\beta_1}{\beta_2}.\tag{22}$$

Professionals on LT&ST know the broad accepted opinion that there is not a direct accordance between coefficients of KM equations and similar coefficients of the RTE. This problem has been discussed for a long time [12–16], and remains still a subject of disputes [10, 17–20]. As the main consequence of simultaneously appeared and, likely, independent publications by Mudgett and Richards [11] and by Brinkworth [15], there is, for example, the well-known result that the relationship between K and S, on the one hand, and μ_a and μ_s, on the other hand, should be written as follows:

$$K\approx 2\mu_a;\ \ S=\frac{3}{4}\mu_s-\mu_a.\tag{23}$$

However, if the second Eq. (23) needs $\mu_a<\frac{3}{4}\mu_s$, that is usually explained as a necessity to have the strong-scattering conditions for the KM approach applicability, the first Eq. (23) in the case of a small scattering looks more dramatically, because it does not contain any dependences on μ_s. In the case of vanishingly small scattering ($\mu_s \to 0$) both KM and 1D RTE equations should have the identical exponential attenuation of light fluxes as the solution of equations. Exponential attenuation cannot differ in two times for the same problem, so, ether $K = \mu_a$ and the first equation (23) is wrong, or K is the unknown function of μ_s with such properties as follows: if $\mu_s = 0$, then $K = \mu_a$, but if $\mu_s \neq 0$, then K aspires to $2\mu_a$ at $\mu_s > \mu_a$. Having Eqs. (17a), (17b), we can give now the more accurate and reasonable answer: $K = \mu_a$ as an electrical property of the non-scattering substance of a medium, $\beta_1 \neq K+S$ and β_1 is a complex function of real optical properties of the scattering media, depending on the mathematical formulation of the problem, $\beta_1-\beta_2\neq\mu_a$ in the general case, etc. The wrong understanding of all this statements leads to errors in numerical calculations.

As the visual example, let us consider the following case of the turbid medium. Let amount of heterogeneities inside the medium is $N = 3$. Also, let $H_0 = 1.5$ cm; $R = 0.4$; and $\mu_a = 0.5$ cm^{-1} [10]. Figure 4 demonstrates forward and backward fluxes computed on the basis of different scattering models. As the exact reference result, we used direct photometric calculations of forward and backward fluxes in the n-layer plane pile [21].

As it follows from the Fig. 4, strictly speaking, none of approximate approaches used describes the exact reference result. Functions of fluxes obeying the reference exact solution are not contiguous function inside the medium, and undergo attenuation jumps at heterogeneities, so finding indefinite derivatives in such breaking points of the first order. As they are not differentiable, they cannot be described by the system of linear differential equations (12). Radiation flux distribution inside the medium in this model has the piecewise continuous (stepwise) character. Therefore, the KM approach has no accurate solution regardless of medium parameters, because this method operates with fluxes having definite first and second derivatives. Nevertheless, if N is large $(N \to \infty)$, any piecewise continuous function would tend to a smooth one. Thus, all approaches above used allow us to describe a smooth approximation of fluxes with their numerical values close to the exact ones on external boundaries of the medium only. This is the theoretical basis of all measurements of transmitted and (or) backscattered fluxes in LT&ST. What is the most important here — it is the fact, that the offered approach with MSA directly inside Δx gives the results, perfectly coinciding with the reference exact solutions on external boundaries of the medium. There are no differences in these quantities at all! Therefore, we can claim that we found out the exemplary exact analytical solution for boundary fluxes using the smooth approximating system (12). Due to the boundary fluxes are the measured quantities, this result is of the great practical interest.

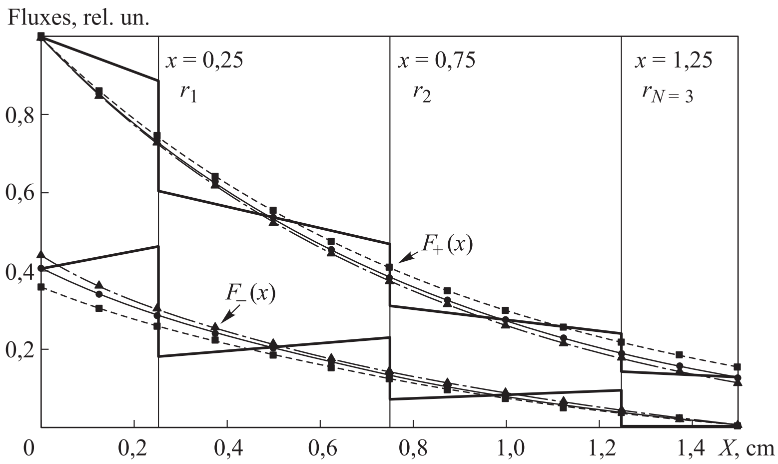

Fig. 4. Forward and backward fluxes computed on the basis of different scattering models (reference exact solution — broken solid line; offered approach based on Eqs. (12), (17a) and (17b) with MSA inside Δx — circles: offered approach based on Eqs. (12) and (16) with SSA inside Δx — rectangles. Mixed approach of independent scattering and absorption with $\beta_1 = \mu_a + \beta_2$ and $\beta_2 = \mu_\rho R/(1-R)$ — triangles)

Revised optical properties. Summation and analysis of all results obtained takes us on the new level of understanding of optical properties definitions and their meaning in LT&ST. It shows that there are a number of incorrect-understandable items and definitions, as, for example, the scattering coefficient (S or μ_s — the notation does not matter here). As we saw, it is not a real optical property of turbid media, but is an effective parameter of approximation models. It differs for MSA and SSA, depending on the presence of absorption inside the medium. At least, four different scattering coefficients — Eqs. (14) and (19) (see the same Eqs. (22) and (36) in the first part of the article [1]), Eq. (11), and Eq. (17b) — have been derived for four different cases of the 1D scattering problem. Which coefficient is correct? The only answer is: all of them, each in its case. Therefore, likely, there is not any sense to select one of them as the original (prime) coefficient. Nevertheless, we would like to suggest the coefficient given by Eq. (14) as the original one. The case of Eq. (19) is too perfect (without absorption) for a practical usage. Cases of Eqs. (11) and (17b) are too complex. Moreover, Eqs. (11) and (17b) contain the scattering coefficient given by Eq. (14) as their part, as well as the scattering coefficient by Eq. (14) forms the exponential attenuation law at SSA (see Eqs. (5), (7) and (16)). Thus, it plays the leading role in a formation of the radiation fields scattered, so it can be considered as an initial scattering coefficient if such a selection has a meaning.

Another parameter is μ_a. In spite of its undeniable role of the absorption coefficient of the medium substance, the fraction of the absorbed radiation within the medium is determined not only by it, but also by the presence of scattering [15]. In most cases of multiple scattering $\beta_1 - \beta_2 \neq \mu_a$. It is true for SSA only. Therefore, if, for example, we solve the problem of induced by external radiation fluorescence inside the turbid medium [21, 22], then at computation of the absorbed part of excitation radiation inside Δx to calculate the fluorescence emission we can get a small error due to $\beta_1 - \beta_2 \neq \mu_a$. Factor

$$\gamma_a = \frac{\beta_1 - \beta_2}{\mu_a} = \frac{\beta_2}{\mu_a}(J - 1) \tag{24}$$

is only a function of R and μ_a / μ_ρ, so it can be presented in relative units. Figure 5 demonstrates possible errors. They can reach 10…12 % in a number of cases. For dielectric biological tissues typical R is 0.02…0.05. In these cases, the error is about a one percent — not so much, but nevertheless.

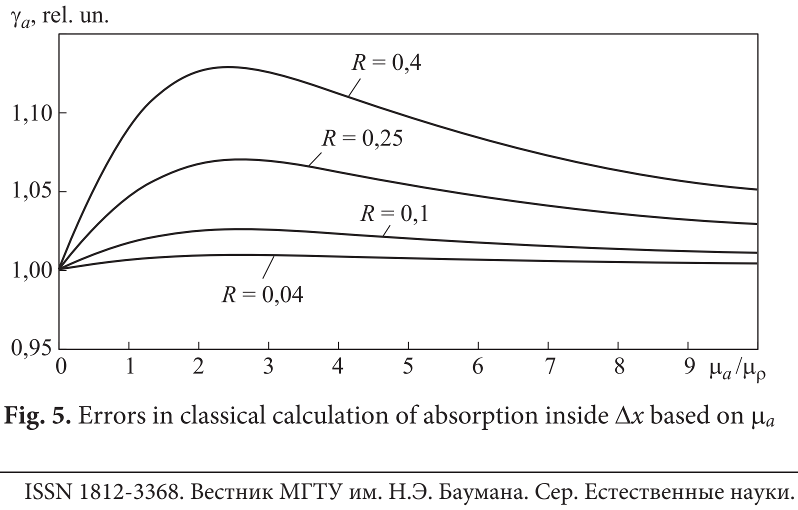

Fig. 5. Errors in classical calculation of absorption inside Δx based on μ_a

Along with it, the single-scattering albedo (W_0) should be considered, as well. Standard definition of the albedo is [8]:

$$W_0 = \frac{\mu_s}{\mu_a + \mu_s}, \tag{25}$$

where $\mu_a + \mu_s$ usually is considered as the attenuation coefficient, i. e., as the first coefficient in the right-hand side of RTE. However, it is not exactly so, in our opinion. One can note, that the equation (25) also can be written as follows:

$$W_0 = \frac{\mu_s}{\left[(\mu_a + \mu_s) - \mu_s\right] + \mu_s}. \tag{26}$$

It means that there are two events: scattering and absorption. Albedo is a fraction of scattering. Scattering is determined by μ_s, i. e., by the second coefficient of differential equations, but absorption is determined by the difference $(\mu_a + \mu_s) - \mu_s$, i. e., by the attenuation coefficient minus the scattering one. Not by pure μ_a. Generally, it corresponds to our $\Delta\beta = \beta_1 - \beta_2$. Therefore, in our approach we have to write:

$$W_0 = \frac{\beta_2}{(\beta_1 - \beta_2) + \beta_2} = \frac{\beta_2}{\beta_1} = \frac{1}{J}. \tag{27}$$

Thus, albedo is the simplest inverse quantity to the Gershun — Gurevich invariant Eq. (21). It is very interesting result! To understand better the difference between classic definition of albedo and Eq. (27), several numeric examples are presented in Fig. 6. We compared W_0 given by Eq. (27) and the following two variants:

$$W_1 = \frac{-\mu_\rho \ln(1-R)}{\mu_a - \mu_\rho \ln(1-R)}; \quad W_2 = \frac{\mu_\rho R/(1-R)}{\mu_a + \mu_\rho R/(1-R)}, \tag{28}$$

which reflect Eq. (26) in different scattering coefficient definitions, close to the classic one.

Once again, we obtained small differences in numerical values depending on original optical properties of the turbid medium. In addition, albedo is broadly used at Monte Carlo simulations to evaluate a probability of scattering. More rigorous definition Eq. (27) can affect the results of the Monte-Carlo statistical computation [23]. In general, these differences are not so dramatic for a practice, but are fundamental for us in a theoretical sense.

Conclusion. The study described in this second part of the article was aimed at finding answers to the problem, how can we use our previously obtained results on the scattering coefficient definition in different cases of scattering with absorption. It was shown, that scattering and absorption processes inside the light-scattering medium are not independent in most cases, so a formulation of the first coefficients of initial differential equations, which mathematically describe the problem, as the simplest

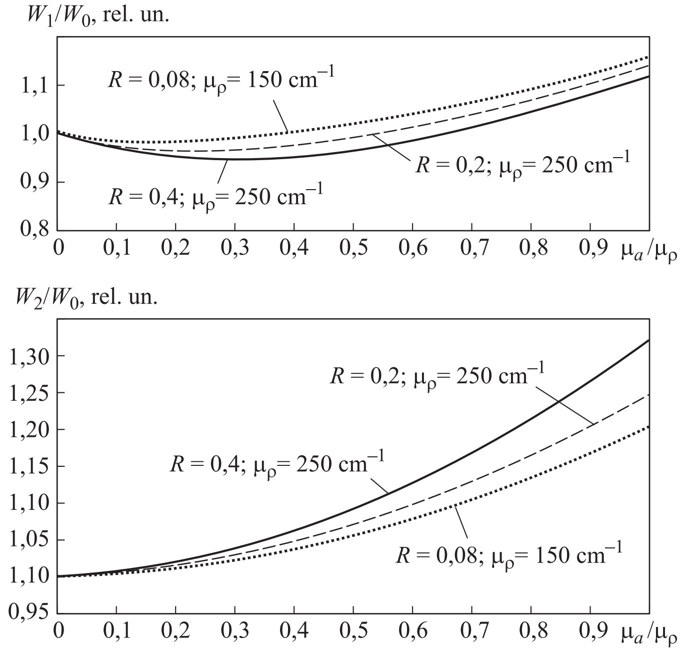

Fig. 6. Ratios of numerical values for different definition of albedo

superposition of scattering and absorption coefficients is wrong. Inaccuracy in this formulations leads to inaccuracies in final results. More correct formulation in application to the classical two-flux Kubelka — Munk (KM) approach, which is a good 1D limit for the radiative transport equation (RTE), allows one to obtain the exact analytical solution for boundary radiant fluxes (backscattered and transmitted ones), contrary to the classic KM approximation. These fluxes are registered by diagnostic equipment in experiments, especially in biomedical applications [24], so this result is very important for the practical usage.

In addition, this result leads to the need for revision of definitions of a number of basic terms in the general radiative transport theory, especially of albedo, which plays a key role in Monte-Carlo simulations. It was obtained in the study, that albedo is the simplest inverse quantity to the Gershun — Gurevich invariant (Eq. (21)) under the correct definition. More rigorous definition for albedo (Eq. (26)) can affect the results of the Monte-Carlo statistical computation. In general, these differences are not so dramatic for a practice, but are fundamental for us in a theoretical sense.

Indeed, much more real and close to realistic practical problems are spatial two-dimensional (2D) or three-dimensional (3D) scattering models. Development of all our ideas and approaches to solve some 2D problems opens also the way to have a new look at several nuances of formulation of the 2D or 3D initial transport equations. We will consider them in the third part of the paper.

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