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THE 2D FLOW AROUND TILTED PLATE

Yu.D. Chashechkin[1] yulidch@gmail.com
Ya.V. Zagumennyi [2] zagumennyi@gmail.com

[1] **Ishlinsky Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow, Russian Federation**
[2] **Institute of Hydromechanics of National Academy of Sciences of Ukraine, Kiev, Ukraine**

Abstract

The 2D problem on stratified flow around a strip is studied numerically and experimentally on the basis of the system of fundamental equations of incompressible viscous fluid mechanics taking into account diffusion effects and neglecting heat transfer. Strongly and weakly stratified fluid flows are considered, as well as potentially and actually homogeneous ones when the buoyancy effects, respectively, are assumed extremely weak or even neglected completely. Initial condition is the diffusion-induced flow on a motionless obstacle, which occurs as a result of interrupting the molecular flux of a stratifying agent on the plate surface. Influence of a set of problem parameters on the flow structure and dynamics is studied, including longitudinal and transverse dimensions of the plate, its shape, angle of attack, and velocity of movement. Analysis of the results shows that the unsteady problem does not have a stationary limit over the entire range of flow parameters due to its internal multiscale structure. The calculated field patterns, being specific for the basic physical variables and their gradients, are in a qualitative agreement with the laboratory data. In the extreme cases, the results converge to the well-known solutions, in particular, to the Blasius solution for the half-plane set in the direction of free stream

Keywords

Stratified and homogeneous fluid, plates with various shapes, angle of attack, wave, vortices, numerical simulation, laboratory experiment

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Introduction. The problem of flow past/around a strip is studied theoretically and experimentally for more than one and half of a century because of the fundamental content and importance of practical applications [1–3], in particular for the development of aircrafts [4]. Experimental studies of the flow past/around an inclined strip conducted in wind tunnels, hydraulic basins and channels [5–7], and more recently in stratified basins [8], allowed to establish important laminar and vortex flow features, to determine the forces and torques in a wide range of parameters.

The complexity of the mathematical description of the flow past a body, which depends on the viscosity and nature of the medium stratification, shape, size, speed of movement and angle of attack of the body, the quality of its surface, the state of the incident flow, allowed the use of approximate methods of analysis [1, 2]. The studies on flow stability [9], boundary layer [2], and turbulence [10] have become widespread among the developed approaches, making it possible to obtain several practically important results for the selected conditions. However, the use of different approaches makes it difficult to compare the results obtained among themselves and experimental data, their extrapolation to new conditions and flow parameters.

The development of computer technology and programming methods made it possible to calculate flows past obstacles based on the system of fundamental equations, which are such derived relations of the “first principles of mechanics” for subsonic [11] and supersonic flows (the fine structure was not visualized [12]). A review of works on analytical and numerical studies of the vortex flow over plate in a wide range of angles of attack is given in [13].

In addition to the traditional studies of the boundary layer and wake past a strip in a homogeneous fluid, the influence of stratification, always existing in the environment and industrial devices, where the fluid density is not constant due to the nonuniformity of the concentration of dissolved substances or suspended particles, temperature and pressure, has been studied during recent years. Fluid particles with different density move vertically along Oz under the action of buoyancy forces, and form a stable stratification $\rho(z)$, that is characterized by scale $\Lambda = |d \ln \rho / dz|^{-1}$, frequency $N = \sqrt{g / \Lambda}$ and buoyancy period $T_b = 2\pi / N$ (g is the gravitational acceleration).

The density variability in the system of equations allows to calculate the dynamics and structure of convective flows and wakes in a single statement within the whole range of velocity values available for study, to conduct a scale invariant classification of structural components, including traditionally studied waves, vortices, and ligaments determining the flow fine structure [14, 15], and to develop methods for direct comparison of data of mathematical (analytical and numerical) and experimental studies of flows [16].

In a quiescent non-equilibrium medium, flows compensating for the substance deficit caused by the interruption of the molecular transfer of a stratifying component appear even near a motionless impermeable obstacle [17]. Laboratory studies have shown that the fine-structured elements of the diffusion-induced flows on a fixed obstacle do not disappear with the onset of motion, but become more complex and thinner. They transform into high-gradient, slowly evolving interlayers separating internal waves, vortices and a wake [8].

Numerical simulation allows to classify flows of continuously stratified fluid in a wide range of obstacle motion parameters and to identify typical flow patterns based on the analysis of their individual structural components, including vortices which are common to all types of fluids, as well as internal waves specific to stratified media, and ligaments, such thin high-gradient interlayers separating regions of typical components, in particular, the wake and internal waves, which are calculated in [18]. A method for determining the vortex layer intensity in solving 2D problems of aerohydrodynamics is proposed in [19], and the general classification of vortex flows is conducted in [20].

In this paper, the 2D problem of vortex flow past a tilted strip has been studied for the first time in coordination with the methods of mathematical and laboratory modeling based on the system of fundamental equations of viscous incompressible isothermal fluid mechanics in order to determine the effect of stratification, longitudinal and transverse dimensions and the shape of the strip (with sharp and rounded edges, straight or narrowed tapering rear part), angle of attack, movement speed. The calculations have been performed in a single formulation in a wide range of parameter values, including creeping, vortex, and nonstationary-vortex regimes up to Re ≈ 100 000.

Mathematical model. The simulation of two-dimensional flows is carried out on the basis of the system of fundamental equations of inhomogeneous incompressible multicomponent fluid mechanics, which includes the equations of state, characterizing the unperturbed density distribution, continuity, momentum and substance transfer in the Boussinesq approximation accounting for the diffusion effects of the stratifying component and neglecting the compressibility, dissipation and heat conduction effects [16],

$$\rho = \rho_{00}\left(\exp(-z/\Lambda) + s\right), \quad \operatorname{div}\mathbf{v} = 0,$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho_{00}}\nabla P + \nu\Delta\mathbf{v} - (s)\mathbf{g}, \quad \frac{\partial s}{\partial t} + \mathbf{v}\cdot\nabla s = \kappa_S \Delta s + \frac{v_z}{\Lambda}. \qquad (1)$$

Here $\rho_{00} = \rho_0(0,0)$ is undisturbed density on the reference level; $s(x, z, t)$ is perturbation of salinity (stratifying component), including the coefficient of salt contraction $\mathbf{v}(x, z, t) = (v_x, v_z)$ is induced velocity; P is pressure excluding the hydrostatic one; $\nu = 0.01$ cm^2/s; $\kappa_S = 1.41 \cdot 10^{-5}$ cm^2/s are coefficients of kinematic viscosity and salt diffusion; t is time; Δ, ∇ are Hamiltonian and Laplacian operator.

The flows of strongly stratified (Λ = 9.8 m, $N = 1.0$ s^{-1}, T_b = 6.28 s) and weakly stratified (Λ = 24 km, $N = 0.02$ s^{-1}, T_b = 5.2 min) media, as well as homogeneous media have been considered in a unified formulation [18]. The media are assumed as *potentially homogeneous* when the buoyancy effects are taken into account in theory, but are not observed experimentally Λ = 10^8 km, $N = 10^{-5}$ s^{-1} and T_b = 7.3 days, and are *actually homogeneous* at $\Lambda = \infty$, $N = 0$ and $T_b = \infty$, when the fluid density is constant.

The calculations and experiments are carried out in two stages. At the first stage, an impenetrable obstacle is introduced into a quiescent stratified medium without any disturbances. On the boundaries of the obstacle, physically reasonable initial and boundary conditions are set, such as no-slip and no-flux conditions on the solid walls, and vanishing of all disturbances at distance from the body in the attached (connected to the body center) coordinate system. At the second stage, diffusion-induced compensation flow, which is formed due to interruption of the molecular transfer of the stratifying component on the obstacle surface [17], is then taken as the initial condition for the flow problem:

$$\begin{gathered} \mathbf{v}\big|_{t\le 0} = \mathbf{v}_1(x, y), \ \ s\big|_{t\le 0} = s_1(x, y), \ \ P\big|_{t\le 0} = P_1(x, y), \ \ v_x\,|_\Sigma = v_z\,|_\Sigma = 0; \\ \left[\frac{\partial s}{\partial \mathbf{n}}\right]\Bigg|_\Sigma = \frac{1}{\Lambda}\frac{\partial z}{\partial \mathbf{n}}, \ \ v_x\,|_{x,z\to\infty} = U, \quad v_z\,|_{x,z\to\infty} = 0, \end{gathered} \tag{2}$$

where U is the uniform free stream velocity at infinity; $\mathbf{n}$ is the outward normal to the strip surface, Σ, with length L and thickness h.

The system of equations (1) and boundary conditions (2) are characterized by a wide range of scales, including large dynamic scales — the internal wavelength $\lambda = UT_b$ and the viscous wave scale $\Lambda_\nu = \sqrt[3]{g\nu} / N = \sqrt[3]{\Lambda\left(\delta_N^\nu\right)^2}$, that reflect the field structure of the attached waves, as well as the universal Stokes, $\delta_N^\nu = \sqrt{\nu / N}$, $\delta_N^{\kappa_S} = \sqrt{\kappa_S / N}$, Prandtl, $\delta_U^\nu = \nu / N$, and Peclet, $\delta_U^{\kappa_S} = \kappa_S / N$, microscales characterizing the flow fine structure. Such a variety of scales of the problem essentially differing in values indicates that the flow past the plate in a general formulation is a complex physical process, accompanied by interactions with each other and with the external flow of simultaneously forming thick and fine structural elements characterized by its own space-time scales, degree of severity and decay rate [14].

Large scales define the minimum dimensions of the observation and calculation areas, which should contain the studied structural elements, including upstream perturbations, wake, waves, vortices, and microscales U determine the cell size and time step. At low speeds of the plate movement U, the Stokes' microscales are critical, while at high velocities, the Prandtl's ones are essential.

The ratios of the basic scales specify dimensionless parameters, including the Reynolds number $\mathrm{Re} = L / \delta_U^{\nu} = UL / \nu$, the Froude number $\mathrm{Fr} = \lambda / (2\pi L) = = U / (NL)$, the Peclet number $\mathrm{Pe} = L / \delta_U^{\kappa_S} = UL / \kappa_S$, sharpness coefficient $\xi_p = L / h$ or form factor $\xi_S = S / (Lh)$, where S is the obstacle cross-sectional area. Additional dimensionless parameters include ratios of scales $C = \Lambda / L$ — an analogue of the inverse Atwood number $\mathrm{At}^{-1} = (\rho_1 + \rho_2) / (\rho_1 - \rho_2)$ for continuously stratified media.

Numerical simulation. The method of numerical solution of the system of equations (1) with the initial and boundary conditions (2) has been developed on the basis of the open source tools *OpenFOAM*. The standard solvers *icoFoam* and *pimpleFoam* developed for modeling unsteady flows of a homogeneous fluid using the finite volume method were supplemented with new variables, including density ρ and salt concentration S, equations and auxiliary parameters, such as buoyancy frequency N and scale Λ, salt diffusion coefficient κ_S, gravitational acceleration g, and others.

In order to ensure the high accuracy of the numerical simulation, the high-precision numerical schemes were used for discretization of the differential equations both in time and space. A *TVD*-scheme with a limiter, which introduces minimal numerical diffusion and ensures absence of oscillations of the solution, was used for interpolation of the convective terms. An implicit three-pointed asymmetric scheme of the second order with backward differencing was applied for discretization of the time derivative, which provides good resolution of the physical process in time. The solution of the resulting system of linear algebraic equations was performed using *PCG* and *PBiCG* iterative solvers, based on the methods of conjugate and bi-conjugate gradients with preconditioning based on the *DIC* and *DILU* procedures, respectively. For coupled calculations of the velocity and pressure fields, a stable well convergent *PISO* algorithm was applied, which shows high efficiency in non-stationary problems. To fulfill the Courant condition, connecting time step Δt with local value of velocity v and size of a computational cell Δr by the ratio $\mathrm{Co} = |v| \Delta t / \Delta r \le 1,$ the *PIMPLE* algorithm was used, which is a kind of the *PISO* algorithm supplemented with a relaxation cycle on values of physical variables at the previous and current time layers, that allows to speed up the calculations due to the possibility of a significant increase of the time step.

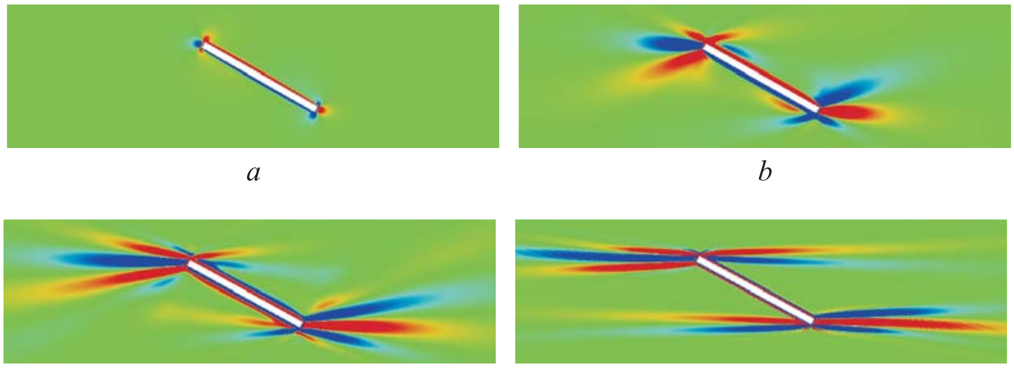
The spatial scales of the computational grid cells are chosen from the condition of adequate resolution of fine-structural flow elements associated with the stratification and diffusion effects, which imposes significant restrictions on the minimum step in space: at least a several calculation cells should be contained in the minimum linear scale in high gradient areas of the flow. The effective and fast generation of an orthogonal computational grid, when the geometric parameters of the problem, such as length, thickness, angle of attack, radius of the plate edge rounding, number of partitions, grid grading in each direction, etc., are changed, has been carried out on the basis of an automated program that allows to recreate the standard control files used by *OpenFOAM* utilities for spatial discretization of the problem. The universal algorithm for generating a computational grid around a plate oriented at an arbitrary angle to horizon consists in constructing two separate computational grids, including a cylindrical mesh area rotating as a whole together with the plate when its angle of inclination is changed and an external fixed mesh area, which are stitched together on the contacting faces using standard utilities of the *OpenFOAM* package.

Due to the high spatial resolution of the problem, the computations were carried out in parallel with the use of high-performance computing resources of the Lomonosov Moscow State University [21] and the federal center for collective use of the NRC “Kurchatov Institute”[*] using the computational domain decomposition method by uniformly dividing the mesh into a certain number of mesh blocks. The developed programs make it possible to carry out calculations of all components of multiscale stratified flows, including upstream perturbations, internal waves, wake and high-gradient separating layers in a wide range of Reynolds numbers, 1 < Re < 100 000, for all types of media in a single formulation.

Results. Interruption of the diffusion flux on a fixed obstacle, immersed in a continuously stratified fluid, leads to salt accumulation on the lower side and

[*] http://ckp.nrcki.ru

to its deficiency on the upper one. This violates the horizontal uniformity of the density distribution and leads to the formation of slow compensation flows, $U \sim 10^{-4}$ cm/s, under the action of the buoyancy forces, directed upwards and downwards along the upper and lower sides of the inclined plate respectively (Fig. 1). At the initial stage of the flow formation, the disturbances are concentrated mainly along the plate surface, while at its edges they are manifested insignificantly (Fig. 1, *a*). The separated jets turn around under the action of the buoyancy forces with their influence increasing with the distance of the separated fluid from the plate edge (Fig. 1, *b*).



a *b* *c* *d*

Fig. 1. Unsteady patterns of the diffusion-induced flow around a fixed tilted plate $N = 1.0$ s^{-1}, $L = 10$ cm, $h = 0.5$ cm, $\alpha = 30°$ at $t = 2$ (*a*), 10 (*b*), 20 (*c*) and 100 s (*d*)

The rotation of the jets and the appearance of circulation cells are accompanied by the formation of compensatory counterflows at the edges of the plate which extend with time, that leads to the complication of the flow pattern (Fig. 1, *c*). For large values of time, the most intense perturbations adjoin the plate edges and extend far beyond the obstacles in the form of extended horizontal layered structures, while the maximum velocities and longitudinal cell sizes continue to increase slowly (Fig. 1, *d*) [17].

With the onset of motion of an obstacle, the pattern of a stratified flow changes dramatically accompanied by formation of upstream perturbations, attached internal wave fields, and a downstream wake past the body [8]. The initial formation of wave disturbances is localized in the region of the sharp edges of the plate, where the most intense processes of fluid displacement and further convergence of the flow components occur (Fig. 2, *a*). A periodic formation of alternating wave disturbances is observed in the structure of the flow, the main generators of which are the upper and lower corners of the front and back edges of the plate (Fig. 2, *b*).

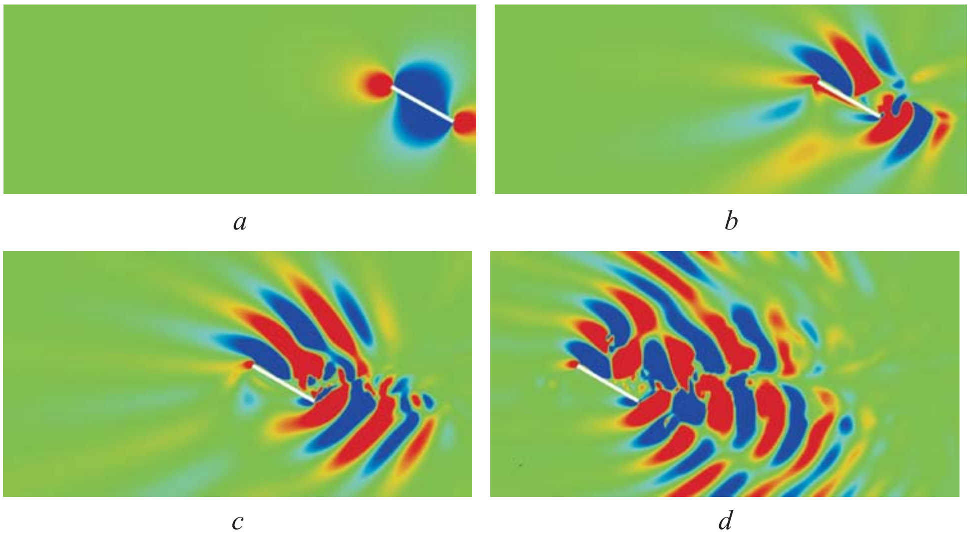


Fig. 2. Unsteady patterns of the stratified flow around a moving tilted plate, $N = 1.0\ \text{s}^{-1}$, $L = 10$ cm, $h = 0.5$ cm, $\alpha = 30°$ at $t = 1$ (*a*), 10 (*b*), 20 (*c*) and 50 s (*d*)

The wave disturbances are localized mainly over the obstacle and in the wake behind it, while the area under the plate remains virtually unperturbed except for the vicinity of the edges (Fig. 2, *c, d*). In the area of the wake, there are consequences of multiple interactions of internal attached waves and tandem vortex structures with the formation of fine-structural flow elements, called ligaments, which are best visualized in the patterns of the horizontal component of the density gradient [14].

The visualization of the unsteady flow pattern with relatively large Reynolds numbers (Re > 10^4), when large and small vortices are simultaneously formed in the vicinity of the front edges is of particular interest [18]. The vortex components actively interact with each other, with thin structures and even with attached internal waves, the length of which significantly exceeds the dimensions of the observation area here. Multiple interactions of different-scale components appear in the nonstationarity of the flow even under fixed boundary conditions. The results of the calculation of the pressure field made for two values of the plate thickness are of scientific and engineering interest (Fig. 3).

The common features of all the figures are a pressure increase in the upstream perturbation in front of the plate, spots of strong rarefaction in the diverging flow at the leading edge, pressure excess on the windward side of the plate and pressure deficit past the leeward side in the areas of localization of vortex elements which are drifting downstream. The vortices are formed on different sides of the plate near its leading edge at low angles of attack, drift along its surface and form a coalesced system, generating wake oscillations with

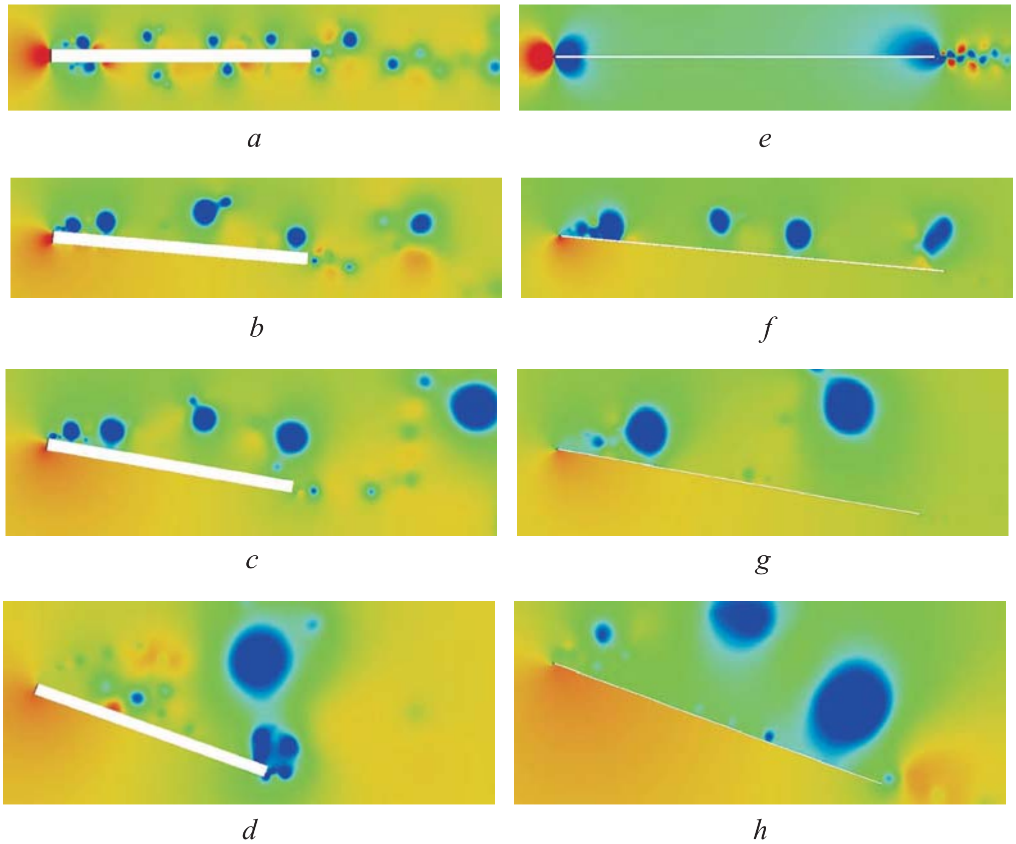
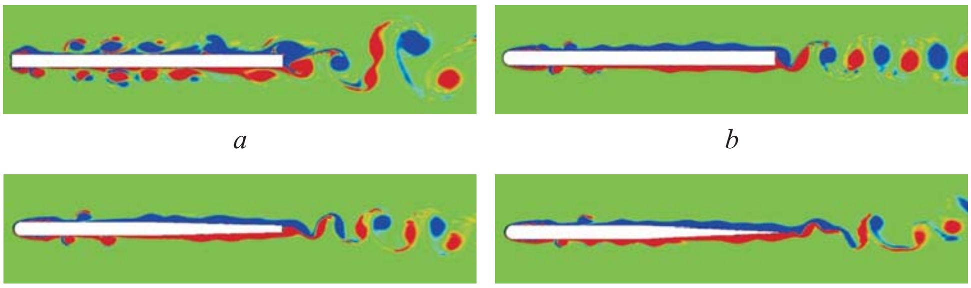


Fig. 3. The structure of the pressure field at different angles of attack of thick $h = 0.5$ cm (*a*–*g*) and thin $h = 0,05$ cm (*e*–*h*) rectangular plates $L = 10$ cm, $N = 1.2$ s^{-1}, $U = 80$ cm/s, $\mathrm{Re} = 80\ 000$ with values $\alpha = 0$ (*a*), 5° (*b*), 10° (*c*), 20° (*d*) (the blue areas correspond to the pressure deficit, the red is to its excess; the scale around the thin plate is increased by 1.5 times)

an almost constant frequency (Fig. 3 *a*). Scales of the vortices and height of the flow region occupied by them grow with an increase in the angle of attack (Fig. 3, *b, c*). At large angles of attack ($\alpha \geq 20°$), the trailing edge of the plate also becomes an additional source for vortices (Fig. 3, *d*).

The size of the vortices is determined by the obstacle thickness at all the other conditions being equal. In the flow on a thick plate, the vortices have clearly outlined cores, while on a thin plate small-scale transverse stripes are formed (Fig. 3 *e*), similar to those observed in the experiment [8]. Here, the size of the vortex bundles also grows with an increase in the angle of attack of the plate (Fig. 3, *f*–*h*). The size of the forming vortices practically does not depend on the plate thickness at angles of attack to the horizon, $\alpha \geq 5°$, and the vortex dynamics is mainly determined by the effective vertical size of the obstacle characterized by the plate length L and the angle of attack, $L\sin\alpha$. At the same time, the differences in unsteady flow patterns around thin and thick plates are determined by the peculiarities of the vortex structure formation in the region of the plate edges.

The instantaneous patterns of the vorticity field are shown in Fig. 4 at the free stream velocity of the stratified fluid, $U = 80$ cm/s, $\mathrm{Re} = 80\,000$, for the three main options of geometrical modification of a horizontal plate: with sharp edges, with a rounded front edge and with a tapering rear part. The vortices drift partially from the region of formation at the leading edge and are independently carried by the flow along the upper and lower edges (Fig. 4, *a*). They come inconsistently to the flow separation zone at the trailing edge. The variability of the position and intensity of converging vortex systems causes deviations of the wake axis from the horizon and oscillations of the position of the wake flow as a whole.



a *b* *c* *d*

Fig. 4. Patterns of the vorticity field at a stratified flow ($N = 1.2$ s^{-1}, $U = 80$ cm/s, $\mathrm{Re} = 80\,000$) over a horizontal plate ($L = 10$ cm, $h = 0.5$ cm):

a with sharp edges; *b* with a rounded front edge; *c, d* with a narrowed tapering rear part (positive values of vorticity (with rotation counterclockwise) correspond to the area of blue color, and negative values (clockwise) — to red color)

The vortex structures drifting along the sides of the plate with a rounded front edge have much smaller scales and intensity, and their centers are localized much closer to the surface of the plate (Fig. 4, *b*). In this case, the region of formation of the vortex structures is shifted downstream by a distance of about two plate thicknesses in contrast to the case of a plate with sharp edges, when the vortices are generated directly at the front edge. The vortices drifting along the sides of the plate retain their identity at a distance of no more than half the plate length from the generation area with their further influence downstream manifesting itself only in the form of damped wave disturbances of the shear layer. In the wake flow region past the plate, the coherent vortex structures are generated with a fairly regular shape and an alternating sign of vorticity.

The unsteady vortex flow structure on the plate with straight sides is quite close to that on the plate with a rounded front edge and tapering rear part (Fig. 4, *c*, *d*). The significant differences are observed only in the wake, where the region of formation of vortex structures past the plate with a pointed trailing edge is significantly narrowed and stretched, and the vortices have smaller scales and are scattered in a significantly larger range of values of the vertical coordinate.

The profiles of the longitudinal velocity component in successive cross sections on thick ($h = 0.5$ cm) and thin ($h = 0.05$ cm) rectangular plates (dashed curves correspond to the Blasius solution for the half-plane [2]) are shown in Fig. 5. The calculated profiles above the thick plate side (Fig. 5, *a*) have

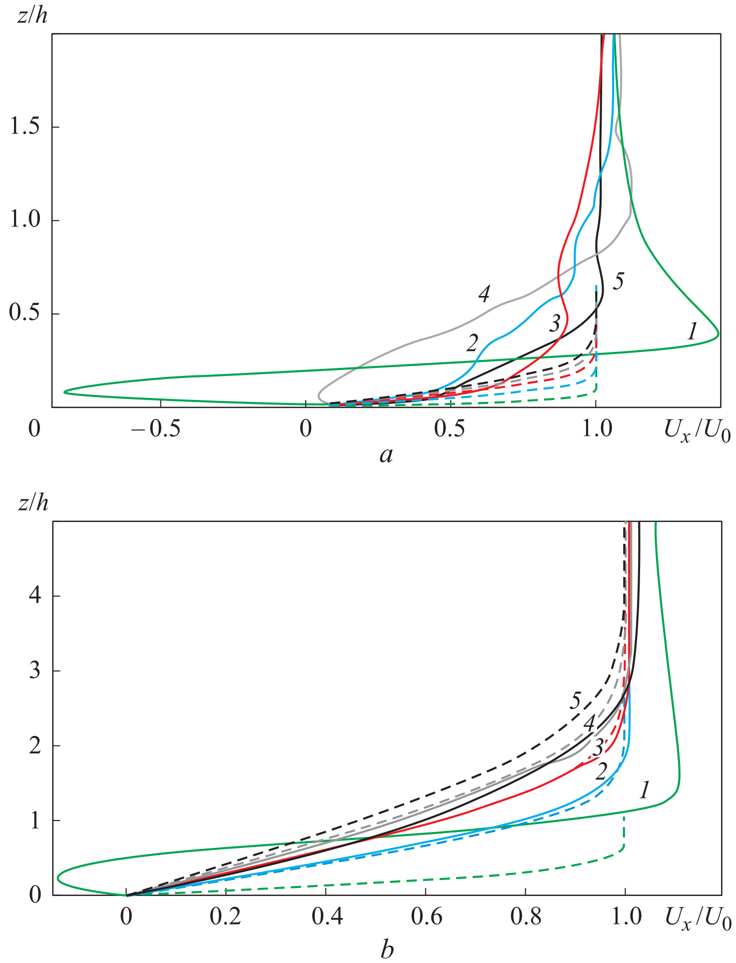
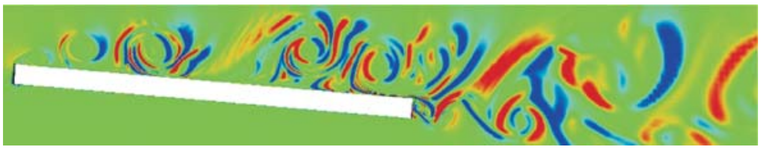


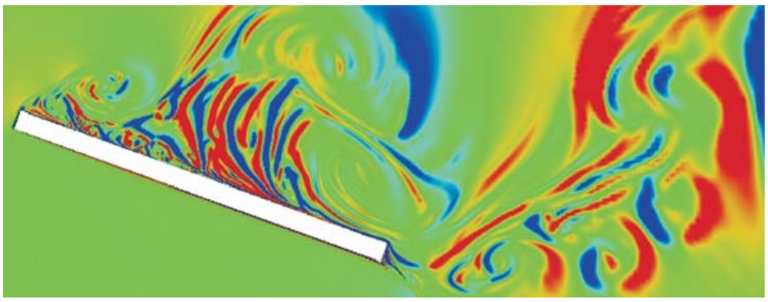
Fig. 5. The instantaneous profiles of the horizontal component of the velocity of the fluid above the plate, streamlined by the flow of a potentially homogeneous fluid ($L = 10$ cm, $N = 10^{-5}$ s^{-1}, $U = 80$ cm/s, Re = 80 000) at various distances from the front edge: *a*, *b* — h = 0.5, 0.05 cm (curves *1*–*5*: $x / L = 0.03$; 0.25; 0.5; 0.75; 1.0)

a rather complicated structure due to the influence of intense vortices and differ significantly along the entire length of the plate from the corresponding profiles obtained from the analytical solution of the boundary layer equations. The good agreement between the calculation results and the theory is observed only in the flow region, which is sufficiently far from the edges of the thin plate (Fig. 5, *b*), while the profiles differ noticeably at the distance less than a quarter of the plate length from its edges.

Each calculated physical variable of the initial system of differential equations (1) reveals some important aspects of the structure and dynamics of a stratified flow, which, in general, contributes to a more complete understanding of the studied physical process. The density gradient fields give additional information about the fine structural flow elements, which are not visible in the fields of other physical variables. The horizontal component of the density gradient, which is shown in Fig. 6 for two different angles of inclination of the



a



b

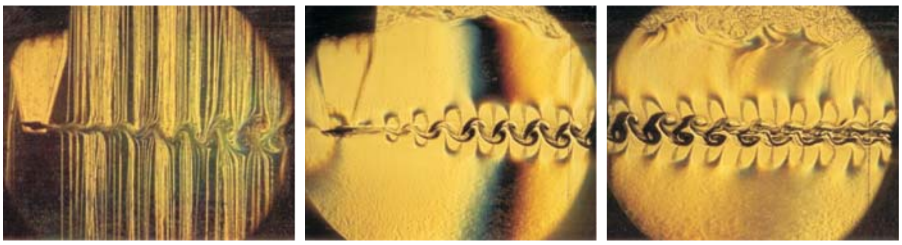
Fig. 6. Instant field patterns of the horizontal density gradient component ($L = 10$ cm, $h = 0.5$ cm, $N = 1.2$ s^{-1}, $U = 80$ cm/s, $\mathrm{Re} = 8\cdot 10^4$ at values of $\alpha = 5°$ (*a*) and 20° (*b*)

plate, is in a linear relation on the light refraction index, such a parameter visualized in laboratory experiments with schlieren instruments. A wide variety of fine structural flow elements is revealed near moving bodies in a stratified fluid with the use of schlieren instruments in laboratory experiments conducted at IPMech RAS, including ligaments, such thin interfaces, envelopes of vortex elements, and fibers [8, 14].

The patterns of the horizontal component of the density gradient field consist of a variety of small-scale multilayer structures of both signs, which are mainly oriented along the streamlines of the vortex flow elements, forming systems of the spiral curls which are typical for vortices. At the same time, small-scale structural elements are localized in flow regions, where active interactions of various flow components between themselves, free stream and rigid boundaries occur. The fineness of the horizontal component of the density gradient field structure can be explained by the smallness of the ratio between the diffusion and the kinematic viscosity coefficients.

The shortest and thinnest structures are localized around the surface on the envelopes of the vortices generated by the front edge of the plate at relatively small angles of inclination of the plate to the horizon (Fig. 6, *a*), and, in the wake flow, the vortex structures become longer and thicker taking much more complex shapes. Due to the growing intensity of interactions between the flow components with the increase in the inclination angle of the moving plate (Fig. 6, *b*), the field geometry becomes much more complicated with enlarging in the number and scale spectrum of typical layered structures above the leeward side of the plate. An important feature of the horizontal component of the density gradient field patterns at $\alpha \geq 20°$ is the formation of multilayer small-scale structural flow elements in the direct vicinity of the plate surface due to intense interactions of large and small-scale components of the vortex flow with the obstacle surface.

The experiments on visualization of the flow pattern past a plate were conducted at the stand "Laboratory Mobile Tank (LMT)" of the Unique Research Facility "Hydrophysical Complex of the IPMech RAS" with high-quality optical portholes using the IAB-458 schlieren instrument [8]. The plate was fixed on thin knives connected to a carriage that moved along guides mounted above the tank. The flow patterns was visualized using a set of markers aligned along the line of thin vertical wakes behind the plunging crystals of salt



a *b* *c*

Fig. 7. The schlieren pattern of the flow behind an inclined plate of $L = 2.5$ cm length, set at an angle of attack $\alpha = 6°$, moving at a velocity of $U = 4.3$ cm/s:
a in a homogeneous and highly stratified fluid; *b*, *c* at $t = 0$, $T_b / 2$, $T_b = 7.5$ s

in a homogeneous fluid (Fig. 7, *a*). A vertical slit and a thin filament in the focal planes of the lighting and receiving parts of the schlieren instrument were used to obtain schlieren patterns in a stratified medium (Fig. 7, *b, c*).

Conclusion. The numerical and experimental studies of the problem on uniform motion of a tilted plate in stratified (strongly and weakly) and homogeneous (potentially and actual) fluids are carried out in 2D formulation on the basis of the system of fundamental equations of fluid mechanics.

The results of calculations and experiments show that the stratified flow around a tilted plate is a complex unsteady process with all the structural components, including waves, vortices, and ligaments, being present simultaneously in varying degrees both at the initial stage of the body movement and in the developed flow regime at large times.

The values of the basic parameters of the medium and the moving body (size, shape, angle of attack, movement velocity) have a significant impact on space-time scales, manifestation degree and attenuation rate of the flow structural elements.

A system of multilayer thin flow elements is formed on the envelopes of vortex elements and in the areas of interaction of different-scale components of the vortex flow with each other and with the obstacle surface, which is visualized in laboratory experiments using schlieren instruments and calculated numerically in the density gradient field structure.

The calculated field patterns, specific for the basic physical variables and their gradients, are in a qualitative agreement with the data of laboratory experiments. In the extreme cases, the results converge to the known solutions, in particular, to the analytical solution of the Blasius problem for the half-plane set along the unperturbed free stream.

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Chashechkin Yu.D. — Dr. Sc. (Phys.-Math.), Professor, Head of Laboratory of Fluid Mechanics, Ishlinsky Institute for Problems in Mechanics, Russian Academy of Sciences (Vernadskogo prospekt 101/1, Moscow, 119526 Russian Federation).

Zagumennyi Ya.V. — Cand. Sc. (Phys.-Math.), Senior Researcher, Hydrobionics and Boundary Layer Control Department, Institute of Hydromechanics of the National Academy of Sciences of Ukraine (Zheliabova ul. 8/4, Kiev, 03057 Ukraine).

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