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APPLICATION OF THE GIBBS MAGNETODYNAMIC PRINCIPLE TO CALCULATION OF THE DISTRIBUTION OF DIRECT CURRENTS IN SOLID BODIES

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Abstract

The thermodynamic hypothesis of Gibbs allowing to solve a problem by means of the magnetic principle of virtual works is applied to finding of equilibrium distribution of superficial and volume stationary currents in a continuous body. The variation of magnetic energy is considered with the additional conditions defining constancy of currents, two of which having a differential appearance are necessary and sufficient for the solution of a task in case of a one-coherent body. If the considered body two-coherent (torus, a thick ring) appears one more condition is necessary. In work it is shown what this condition which is also providing uniqueness of the decision can be or constancy of the current proceeding through cross section a torus, or a task of a constant stream of magnetic induction through an opening a torus. At problem definition the first option as more evident was chosen. The problem is solved with the help of a method of Lagrange multipliers. The main received result is that circumstance that induction of magnetic field and volume current in volume address in zero. Thus, magnetic field together with currents is squeezed out on a surface. Communication of the received results with Meissner — Ochsenfeld effect and the London's equation applied in the theory of superconductivity and also a problem of communication of molecular currents and currents of conductivity are discussed

Keywords

Volume currents, superficial currents, two-dimensional vector analysis, variation analysis, Lagrange multipliers, stream of magnetic induction, ideal magnetic, London's equation, Meissner — Ochsenfeld effect

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Introduction. The structure of such difficult system as magnetic properties of materials constantly is for a long time in sight of researchers, since the earliest works, such as G. Green (1871) [1] — the section "application of the preliminary results to theory of magnetism" and K.W. Thomson (1872) [2] in which the majority of the results known by then till present devoted to the considered subject [3–5] were collected. And, despite huge progress in practical use of magnetism (from compasses to computer memory), the nature of this complex structure remains not clear till present. It is enough to mention the problem connected with the fact that still it is not possible to reduce to uniform model Coulomb laws and Biot— Savart who is the cornerstone of phenomenological approach to magnetism though it is clear that the nature of this phenomenon, naturally, the general [6].

In the real work the question of the nature of distribution of volume and superficial direct currents is investigated and also — characteristics of magnetic field in a continuous solid body.

In our opinion, the statement and the solution of this kind of tasks promotes further advance and expansion in the field of understanding of bases of magnetism and use of the corresponding theoretical achievements in applied technical problems.

Calculation of a variation of magnetic energy of the two-coherent structure which is in the thermostat. For a conclusion of a variation of magnetic field we will consider topological two-coherent body (torus) of volume V, with a surface S on which volume and superficial currents of density proceed $\vec{j}$ and $\vec{i}$, and being in the thermostat with characteristics V_0, T_0, p_0 (volume, temperature and pressure in the thermostat respectively). Ideally carrying out cover S_0 shields magnetic field. Volume and superficial external forces are set $\rho\vec{f}$, $\vec{F}$ acting on a body. Let's find an equilibrium condition of a body and we will receive a full set of necessary and sufficient conditions of balance of a body — thermal, mechanical and magnetic (Fig. 1).

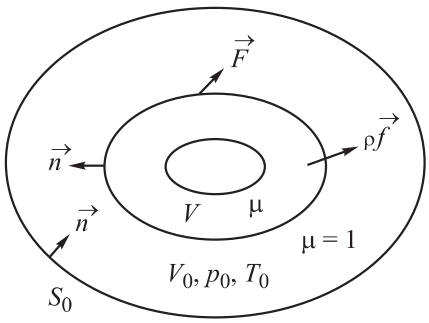

Fig. 1. The scheme to calculation of energy of magnetic field

The general regional task comes down to the system of the equations:

$$\operatorname{rot}\left(\frac{1}{\mu_0\mu}\operatorname{rot}\vec{A}(\vec{r})\right)=\vec{j}(\vec{r})\quad \operatorname{rot}\vec{A}=\vec{B}\qquad \vec{r}\in V;$$

$$\operatorname{rot}\left(\operatorname{rot}\vec{A}_0\left(\vec{r}\right)\right)=0 \quad \operatorname{rot}\vec{A}_0=\vec{B}_0 \quad \vec{r}\in V_0,$$

where $\vec{A}$ is the vector potential of magnetic field.

Additional condition is Coulomb calibration of vector potentials:

$$\operatorname{div}\vec{A}=0;$$
$$\operatorname{div}\vec{A}_0=0.$$

It is known that this calibration is applied by consideration of the nonrelativistic magnetostatics tasks in difference from Lorentz's calibration applied to consideration of dynamic tasks.

Boundary conditions have an appearance

$$A_n\Big|_S= A_{0n}\Big|_S; \ \left[\left(\frac{1}{\mu}\operatorname{rot}\vec{A}-\operatorname{rot}\vec{A}_0\right),\ \vec{n}\right]=\mu_0\vec{i},$$

where $\vec{A}_0\big|_{S_0}=0$ is condition of ideal conductivity of a cover S_0.

To exclude variations of unknown sizes $\delta\vec{A}$ and $\delta\vec{A}_0$, we will calculate energy and, respectively, its variation in two various ways

$$W_1=\frac{1}{2}\int\limits_V\frac{B^2}{\mu_0\mu}dV+\frac{1}{2}\int\limits_{V_0}\frac{B_0^2}{\mu_0}dV;\ W_2=\frac{1}{2}\int\limits_V\left(\vec{j},\vec{A}\right)dV+\frac{1}{2}\oint\limits_S\left(\vec{i},\vec{A}\right)dS;$$
$$W=W_1=W_2;\quad \delta W=2\delta W_2-\delta W_1.$$

Transformations lead to the following expression for a variation of magnetic field

$$\delta W=\frac{1}{2\mu_0}\int\limits_S\left\{\left(\frac{B^2}{\mu}-B_0^2\right)\delta^* q_n-\frac{2}{\mu}B_{0n}\left(\vec{B},\delta^*\vec{q}\right)+2B_{0_n}\left(\vec{B}_0,\delta^*\vec{q}\right)\right\}dS+$$
$$+\frac{1}{2\mu_0}\int\limits_V\frac{B^2}{\mu^2}\delta\mu\,dV+\int\limits_V\left(\delta^*\vec{q},\left[\vec{j},\vec{B}\right]\right)dV+\int\limits_V\left(\delta\vec{j},\vec{A}\right)dV+\oint\limits_S\left(\delta^*\vec{i},\ \vec{A}_0\right)dS.$$

Here q is shift; $\vec{q}$ is the vector of shift; $\delta^*\vec{q}$ are variation of shift; $\delta\vec{j}$, $\delta^*\vec{i}$ are these are variations of density of free currents. At the same time, we will note two circumstances. "Frozen" currents were excluded, there were only free currents. Variations of volume currents $\delta\vec{j}$ are written out in a usual look through Euler coordinates, and variations of superficial currents $\delta^*\vec{i}$ are written out in Lagrange variables. The matter is that in Euler coordinates of coordinate of observation also all happening changes are rigidly fixed, including also variations, kind of "pass" by these points, remaining in body borders. But the boundary surface is displaced at variation, and together with it "leave aside" and superficial currents, passing observation points. Therefore, more in physically as it seems to us, will tie superficial currents to deformable border, as is an essence of Lagrange variables. The similar remark belongs also to $\delta^* \vec{q}$ to variations of shifts.

Thermodynamic Gibbs principle. So, the isolated system a body thermostat for that simple reason is considered that for it the known thermodynamic principle of Gibbs allowing to find in general any conditions of balance, in any physical and physical and chemical situations is fair.

Further the received result is used for record of the magnetic principle of virtual works. The only necessary and sufficient condition of balance, the following from the magnetic principle of virtual works, has an appearance:

$$\delta\left(U - T_0 S + p_0 V - W\right) = \int_V \rho\left(\vec{f}, \delta^* \vec{q}\right) dV + \oint_S \left(\vec{F}, \delta^* \vec{q}\right) dS,$$

where $\vec{F}$ is the vector of superficial external force; and last composed in the left part, apparently, it is more correct to connect not with the principle of conservation of energy and the principle of a minimum of energy following from it, and with the principle of the smallest action. Except internal energy U and composed $T_0 S$ and $p_0 V$, considering conditions under which in the thermostat constant conditions are supported (T_0, p_0), under the sign of a variation the total energy of magnetic field enters W. It is essential that it enters with a minus sign, but not with a plus, as in case of electric field. Variations of entropy and volume register as usual:

$$\delta S = \int_V \rho \delta^* s dV; \quad \delta V = \oint_S \delta^* q_n dS.$$

Then, conditions of balance of a firm magnetic with "frozen" currents are as follows:

a) thermal balance $T = T_0$, where

$$T = \left(\frac{\partial u}{\partial s}\right)_q - \frac{B^2}{2\mu_0 \mu^2 \rho}\left(\frac{\partial \mu}{\partial s}\right)_q$$

— determination of local temperature;

b) balance on a surface $F_i = Q_{im} n_m - Q^0{}_{im} n_m$, where full tensor of tension $Q_{im} = P_{im} + T_{im}$ it consists of a tensor of internal mechanical tension

$$P_{im} = p_{im} - \frac{B^2}{2\mu_0 \mu^2} \frac{\partial \mu}{\partial q_{nm}} \left(\delta_{ni} - q_{ni}\right),$$

and a Maxwell's tensor of tension

$$T_{im} = -\frac{1}{2\mu_0}\frac{B^2}{\mu}\delta_{im} + \frac{1}{\mu_0\mu}B_m B_i;$$

c) balance in volume

$$\frac{\partial Q_{im}}{\partial x_m} + \rho f_i = 0.$$

Thus, from the only necessary and sufficient condition of the balance following from the magnetic principle of virtual works considered by us there are in the beginning necessary and sufficient conditions of thermal and mechanical balance which will coincide with similar in case of “frozen” currents as for a case of liquid, and firm magnetics. Then in the basic equation of the magnetic principle of virtual works will remain only composed, connected with free currents. Other composed led to balance conditions taking into account “frozen” currents.

Method of Lagrange multipliers. So, the case when is considered: $\delta\vec{j} \neq 0$, $\delta^* i \neq 0$, nevertheless other composed, connected with an entropy variation, and variations of vectors of shift, according to the main technique of a variation method, nullifying.

The variation of magnetic energy in this case takes a form:

$$\delta W = \int_V \left(\vec{A}, \delta\vec{j}\right) dV + \oint_S \left(\vec{A}, \delta^* \vec{i}\right) dS. \tag{1}$$

For the solution of a variation task it is necessary to consider also additional conditions, two of which (conditions of constancy of currents) have differential character,

$$\operatorname{div}\vec{j} = 0, \quad j_n = \operatorname{Div}\vec{i} \tag{2}$$

(here under $\vec{n}$ the external normal to a surface of the considered body), and one — integrated is understood (a condition of constancy of the total current proceeding through any cross section σ torus and curve L, limiting this section, Fig. 2):

$$I = \int_\sigma \vec{j} d\vec{\sigma} + \oint_L i_\tau dl = \text{const}, \tag{3}$$

where i_τ is projection of superficial current to the direction, perpendicular dl — to a curve element L and to an element dS surfaces. Let’s note one important circumstance. For one-coherent conductors the value of a magnetic flux through an opening a torus and communication which with magnetic characteristics, it will be discussed below there are enough conditions (2) whereas for two-coherent (torus, a thick ring, see Fig. 1) needs one more condition presented in the form (3), fixing value of a total current or as it will be shown.

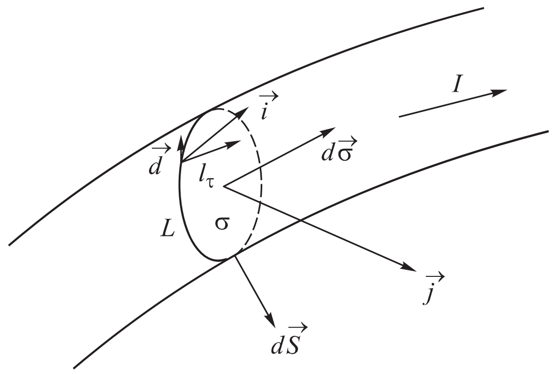

Fig. 2. The scheme illustrating a condition of constancy of a total current

The second condition from (2) expresses that circumstance that the volume current flowing into this place on a surface spreads then on a surface in the form of superficial current ($\mathrm{Div}\,\vec{i}$ is superficial divergence of a vector $\vec{i}$).

So, we will formulate the variation principle for search of equilibrium distribution of current of free charges. The variation condition consists that the variation of energy (1) has to equal to zero at any variations of currents $\delta\vec{j}$, $\delta^*\vec{i}$, meeting additional conditions (2), (3). For account at variation of these conditions we use a method of multipliers of Lagrange according to which it is possible to reject in general them if to equate to zero not a variation of initial size energy, but some combination of sizes.

At record of a method of Lagrange multipliers we will consider that circumstance that we have to write down a contribution from differential composed for all points of a body and, therefore, Lagrange multipliers $\alpha(\vec{r})$ and $\beta(\vec{r})$ are the multiple-valued functions set in the volume of V and on S surface respectively whereas ς number:

$$\mathrm{div}\,\vec{j}=0\Rightarrow\alpha(\vec{r});\quad \mathrm{Div}\,\vec{i}-j_n=0\Rightarrow\beta(\vec{r});\quad \int_\sigma \vec{j}d\vec{\sigma}+\oint_L i_\tau dl-I=0\Rightarrow\varsigma.$$

Then record of the magnetic principle of virtual works takes a form

$$\delta W+\delta\int_V \alpha(\vec{r})\,\mathrm{div}j(\vec{r})\,dV+\delta\int_S \beta(\vec{r})\left(\mathrm{Div}\,\vec{i}-j_n\right)dS+$$

$$+\delta\left[\varsigma\left(\int_\sigma \vec{j}\,d\vec{\sigma}+\oint_L i_\tau dl-I\right)\right]=0.$$

As volume V let's consider the part a torus limited to two cross sections σ_p and σ_q (here under n the normal to a surface of the cross section is understood, Fig. 3).

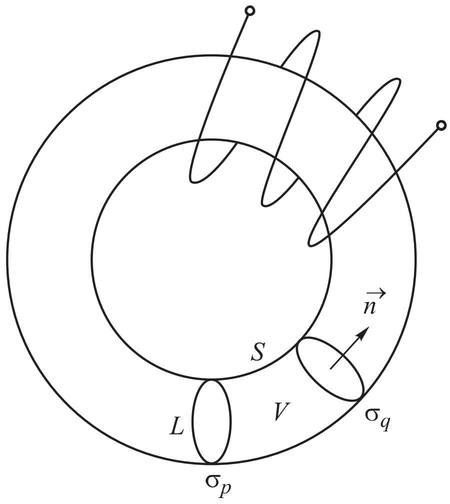

Fig. 3. The scheme illustrating jumps of Lagrange multipliers

No restrictions for the chosen volume are imposed, in particular the specified sections can be very close to each other. In this case volume will actually coincide with the volume of all a torus. Respectively under S the side surface of volume is understood V.

As variation goes on variables of free currents,

$$\int_V (\vec{A},\delta\vec{j})\,dV + \int_S (\vec{A},\delta^*\vec{i})\,dS + \int_V \alpha(\vec{r})\,\mathrm{div}\,\delta\vec{j}\,dV + \int_S \beta(\vec{r})\left(\mathrm{Div}\,\delta^*\vec{i} - \delta j_n\right)dS +$$
$$+\varsigma\int_\sigma \delta j_n\,d\sigma + \varsigma\oint_L \delta^* i_\tau\,dl = 0;$$

$$\int_V (\vec{A},\delta\vec{j})\,dV + \int_S (\vec{A},\delta^*\vec{i})\,dS + \varsigma\int_\sigma \delta j_n\,d\sigma + \varsigma\oint_L \delta^* i_\tau\,dl =$$
$$= -\int_V \left[\mathrm{div}(\alpha\delta\vec{j}) - (\delta\vec{j},\nabla\alpha)\right]dV - \int_S \left[\mathrm{div}(\beta,\delta^*\vec{i}) - \delta^*\vec{i}\,\mathrm{Grad}\,\beta\right]dS +$$
$$+\int_S \beta(\vec{r})\,\delta j_n\,dS = -\left\{\int_S \alpha\delta j_n dS + \int_\sigma (\alpha_p - \alpha_q)\,\delta j_n\,d\sigma\right\} +$$
$$+\int_V (\delta\vec{j},\nabla\alpha)\,dV - \oint_L (\beta_p - \beta_q)\,\delta^* i_\tau\,dl + \int_S \delta^*\vec{i}\,\mathrm{Grad}\,\beta\,dS + \int_S \beta(\vec{r})\,\delta j_n\,dS.$$

At calculation of integral by means of Gauss — Ostrogradsky's theorem we passed from volume V to the closed surface consisting from σ_p, σ_q, S.

Stokes's theorem is the same way applied. Finally,

$$\int_V \left((\vec{A} - \nabla\alpha),\delta\vec{j}\right)dV + \int_S \left((\vec{A} - \mathrm{Grad}\,\beta),\ \delta^*\vec{i}\right)dS + \int_S (\alpha - \beta)\,\delta j_n\,dS +$$
$$+\int_\sigma (\alpha_p - \alpha_q - \varsigma)\,\delta j_n\,d\sigma + \oint_L (\beta_p - \beta_q - \varsigma)\,\delta^* i_n\,dl = 0,$$

where $\mathrm{Grad}\,\beta$ is superficial gradient of scalar function $\beta(\vec{r})$.

Further we will apply the standard procedure of calculus of variations.

1. At $\delta\vec{j} \neq 0$ in volume V

$$\vec{A} - \nabla\alpha = 0 \quad \vec{A} = \nabla\alpha \quad \vec{B} = \operatorname{rot}\vec{A} = \operatorname{rot}\nabla\alpha = 0, \text{ therefore, } \vec{j} = 0.$$

Let's show that in our case Lorentz's calibration $\operatorname{div}\vec{A} = 0$ the equation is carried out and will receive for $\alpha(\vec{r})$. Really, according to definition, magnetic scalar potential has an appearance

$$\vec{A} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{j}(\vec{r}')}{R}dV' + \frac{\mu_0}{4\pi}\oint_S \frac{\vec{i}(\vec{r}')}{R}dS',$$

where $\vec{R} = \vec{r}' - \vec{r}$, $\vec{r}'$ is radius — a vector with the corresponding density of current, $\vec{r}$ is observation point radius vector, and, therefore,

$$\operatorname{div}\vec{A} = \operatorname{div}_r \frac{\mu_0}{4\pi}\int_V \frac{\vec{j}(\vec{r}')}{R}dV' + \operatorname{div}_r \frac{\mu_0}{4\pi}\oint_S \frac{\vec{i}(\vec{r}')}{R}dS' =$$

$$= \frac{\mu_0}{4\pi}\int_V \operatorname{div}_r \frac{\vec{j}(\vec{r}')}{R}dV' + \frac{\mu_0}{4\pi}\oint_S \operatorname{div}_r \frac{\vec{i}(\vec{r}')}{R}dS' =$$

$$= \frac{\mu_0}{4\pi}\int_V \vec{j}(\vec{r}')\nabla_r \frac{1}{R}dV' + \frac{\mu_0}{4\pi}\oint_S \vec{i}(\vec{r}')\nabla_r \frac{1}{R}dS' =$$

$$= -\frac{\mu_0}{4\pi}\int_V \vec{j}(\vec{r}')\nabla_{r'} \frac{1}{R}dV' - \frac{\mu_0}{4\pi}\oint_S \vec{i}(\vec{r}')\nabla_{r'} \frac{1}{R}dS' =$$

$$= -\frac{\mu_0}{4\pi}\int_V \left[\operatorname{div}_{r'}\left(\vec{j}(\vec{r}')\frac{1}{R}\right) - \frac{1}{R}\operatorname{div}_{r'}\vec{j}(\vec{r}')\right]dV' -$$

$$- \frac{\mu_0}{4\pi}\oint_S \left[\operatorname{Div}_{r'}\left(\vec{i}(\vec{r}')\frac{1}{R}\right) - \frac{1}{R}\operatorname{Div}_{r'}\vec{i}(\vec{r}')\right]dS' =$$

$$= -\frac{\mu_0}{4\pi}\oint_S \left(j_n(\vec{r}')\frac{1}{R}\right)dS' - \frac{\mu_0}{4\pi}\oint_S \left(\frac{1}{R}\operatorname{Div}_{r'}\vec{i}(\vec{r}')\right)dS' = 0.$$

The last ratios are received taking into account conditions of isolation (3):

$$\operatorname{div}_{r'}\vec{j}(\vec{r}') = 0, \ \operatorname{Div}_{r'}\vec{i}(\vec{r}') = j_n(\vec{r}'),$$

and also obvious equality $\nabla_r \frac{1}{R} = -\nabla_{r'}\frac{1}{R}$ and properties of superficial divergence: $\oint_S \operatorname{Div}\frac{\vec{i}(r')}{R}dS' = 0$. Then $0 = \operatorname{div}\vec{A} = \operatorname{div}\operatorname{grad}\alpha = \Delta\alpha$, i.e., $\alpha(\vec{r})$ in a body satisfies to Laplace's equation.

In volume there is no field and, therefore, there are no volume currents. Currents are forced out on a surface. The received result has a simple physical explanation. Really, from the continuity equation taking into account stationarity of a task: $\frac{\partial \rho}{\partial t} = 0$ follows $0 = \operatorname{div} \vec{j} = \operatorname{div} \sigma \vec{E} = \operatorname{div} \frac{\sigma}{\varepsilon \varepsilon_0} \vec{D}.$ Taking into account Gauss theorem follows from the last ratio that $\rho = 0.$ Thus, in case of the stationary mode macroscopic electric charges, and, therefore, and the related free currents can be in a uniform magnetic (the carrying-out environment) only on a surface.

2. At $\delta j_n \neq 0$ on a surface $S: \alpha = \beta$ in section σ:

$$\langle \alpha \rangle = \alpha_p - \alpha_q = \varsigma. \tag{4}$$

3. At $\delta^* \vec{i} \neq 0.$

3.1. On a surface $S: \vec{A} = \operatorname{Grad} \beta$ from where follows that a vector of magnetic potential collineated $\operatorname{Grad} \beta,$ which, in turn, is directed on a tangent to a surface, i.e., $A_n = 0$ and, respectively, according to the previous point: $\left. \frac{\partial \alpha}{\partial n} \right|_S = 0.$

3.2. Along a curve L:

$$\langle \beta \rangle = \beta_p - \beta_q = \varsigma. \tag{5}$$

From (4) and (5) follows that jumps of multiple-valued functions α and β on σ do not depend on a form of a curve in which all two-coherent area since manages.

So, free current has superficial character and for search of its density the regional problem is solved: it is necessary to find function $\alpha(\vec{r}),$ which in a body satisfies to Laplace's equation: $\Delta \alpha = 0|_V$ and to a boundary condition $\left. \frac{\partial \alpha}{\partial n} \right|_S = 0.$ There is open a question of jump of this function at a round (4).

The theorem about uniqueness of the received decision. Let's consider circulation of a vector $\vec{A}$ on any contour $M,$ then taking into account the ratios received above we receive

$$\oint_M \vec{A} d\vec{l} = \oint_M \left(\nabla \alpha, d\vec{l} \right) = \alpha_p - \alpha_q \equiv \langle \alpha \rangle = \varsigma.$$

On the other hand

$$\oint_M \vec{A} d\vec{l} = \int_F \left(\operatorname{rot} \vec{A} \right) d\vec{S} = \int_F \vec{B} \, d\vec{S} = \Phi,$$

where Φ is a stream of magnetic induction; σ is the surface leaning on L. Therefore, the magnetic flux through any surface leaning on a curve inside a torus is identical and coincides with Lagrange multiplier: $\Phi = \varsigma$.

Considering that function $\alpha(\vec{r})$ according to (4) at a round experiences jump, there is a question of unambiguity of the received decision. For this purpose, we will remove in the beginning an auxiliary ratio which can be treated as a certain theorem. The field out of a torus (volume $\tilde{V}$) does not contain currents. Then, according to the equation $\operatorname{rot}\vec{B}(\vec{r})=0$ $(\vec{r}\in\tilde{V})$ there is a magnetic scalar potential ψ, such that $\vec{B}=-\nabla\psi$. On a torus current flows I (3) for which we will write down the theorem of circulation on any almost closed contour l (Fig. 4) resting against two very close points P and Q the areas located on both sides from a membrane Σ tightening an opening a torus: $\int\limits_{P(l)}^{Q}\vec{B}d\vec{l}=\mu_0 I=\int\limits_{\sigma}\left(\operatorname{rot}\vec{B}\right)d\vec{S}.$

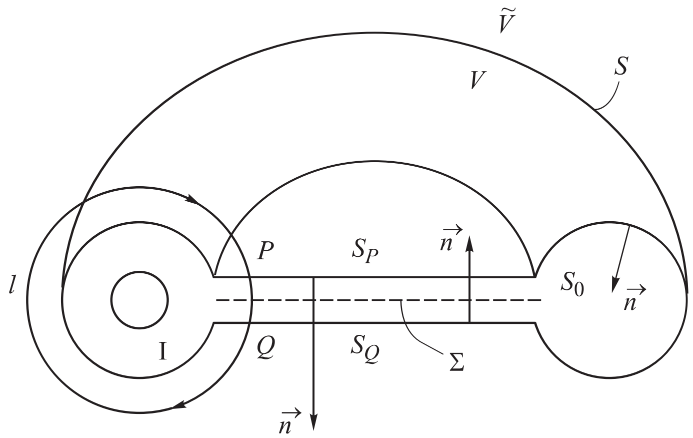

Fig. 4. The scheme illustrating a conclusion of uniqueness of the decision

Curve l occupies the square big, than cross section a torus σ but in other points $\operatorname{rot}\vec{B}=0$ and therefore this transition is lawful.

Further,

$$\mu_0 I=-\int\limits_{P(l)}^{Q}\nabla\psi d\vec{l}=-\int\limits_{P(l)}^{Q}\frac{d\psi}{dl}dl=\psi(P)-\psi(Q). \qquad (6)$$

Let's write down energy of magnetic field out of a torus $R=\int\limits_{\tilde{V}}\frac{B^2}{2\mu_0}dV$ also we will transform it taking into account that $\operatorname{div}\vec{B}=0$:

$$R=\frac{1}{2\mu_0}\int_{\tilde{V}}\vec{B}\left(-\nabla\psi\right)dV=\frac{1}{2\mu_0}\int_{\tilde{V}}\left(-\operatorname{div}\left(\vec{B}\psi\right)+\psi\operatorname{div}\vec{B}\right)dV=-\frac{1}{2\mu_0}\int_{\tilde{V}}\operatorname{div}\left(\vec{B}\psi\right)dV.$$

Further transformation is connected with Gauss's theorem. Generally speaking, the closing surfaces at volume $\tilde{V}$ four: the surface S of torus, two flat surfaces S_P and S_Q, passing through points P and Q, torus, close to the plane of overlapping of an opening, and infinitely far surface, on which it is natural to put $\vec{B}=0$ and therefore the contribution from it to Gauss theorem will not be considered. Then, taking into account the sign of a normal which is directed everywhere from $\tilde{V}$:

$$R=\frac{1}{2\mu_0}\int_{S}\psi B_n dS+\frac{1}{2\mu_0}\int_{S_P}\psi_P B_n dS-\frac{1}{2\mu_0}\int_{S_Q}\psi_Q B_n\, dS.$$

Further we consider that B_n on limits of the section it is continuous. Then from this condition and value $\vec{B}=0$ inside a torus we receive that $B_n\big|_S=0,$ and

$$R=\frac{1}{2\mu_0}\int_{\tilde{S}}\left(\psi_P-\psi_Q\right)B_n dS=\frac{1}{2\mu_0}\left(\psi_P-\psi_Q\right)\int_{\tilde{S}}B_n\, dS.$$

Finally, taking into account (6) we receive: $R=\frac{1}{2}I\Phi,$ where Φ is a stream of magnetic induction through an opening a torus; I is total current in a torus.

We use the received result for a research of a question of uniqueness of the solution of the studied task. Let us have two solutions of distribution of magnetic field in $\tilde{V}$: $\vec{B}_1$ and $\vec{B}_2$. Let's consider the field combined from this field in a look $\vec{B}_0=\vec{B}_1-\vec{B}_2$. Owing to the theorem proved above it is possible to write down

$$\int_{\tilde{V}}\frac{B_0^2}{2\mu_0}dV=\frac{1}{2}I_0\Phi_0, \tag{7}$$

where owing to linearity of a task $I_0=I_1-I_2$ and $\Phi_0=\Phi_1-\Phi_2$. Indexes 1 and 2 belong, respectively, to the first and second version of the decision. The right part of the ratio (7) turns into zero in one of two options:

1) integrated current (3) as task parameter which unambiguously defines circulation of a vector of magnetic field is set. In this case, $I_0=I_1-I_2=I-I=$ $=0;$

2) the stream of magnetic induction through an opening a torus is set Φ. In this case, $\Phi_0=\Phi_1-\Phi_2=\Phi-\Phi=0.$

In any of these cases $\int_{\tilde V} \frac{B_0^2}{2\mu_0} dV = 0$ from where follows that $B_0^2 = 0$ (owing to randomness of the chosen volume $\tilde V$), and as $B_0 = B_1 - B_2 = 0,\ B_1 = B_2$. Thus, for two-coherent area in the presence of one of two additional conditions the decision out of a torus turns out only. The given reasons show that in case of one-coherent bodies it is not necessary to impose any additional conditions, the Decision already is only.

Results. So, induction and volume currents in a body are equal to zero. Free current has superficial character and for search of its density the regional problem is solved: it is necessary to find multiple-valued function $\alpha(\vec r)$, which inside torus V satisfies to Laplace's equation: $\Delta\alpha = 0|_V$ and to a boundary condition $\left.\frac{\partial\alpha}{\partial n}\right|_S = 0,$ and the jump of function at a round is set and equal $\langle\alpha\rangle = \varsigma = \Phi.$ In volume of the thermostat surrounding a body respectively: $\Delta\vec A_0 = 0|_{V_0},\ \vec A_0 = 0|_{S_0},\ A_{0n} = 0|_S.$ Knowing $\vec A_0$ and α it is possible to define $\vec B_0 = \operatorname{rot}\vec A_0$ and on it distribution of superficial currents: $\vec i = \frac{1}{\mu_0}\left[\vec n, \vec B_0\right].$

The problem is solved at set Φ also has the only decision. Let's notice one important circumstance: at deformations of a ring-shaped body Φ remains invariable. This result was well-known in case of a body with ideal conductivity. The matter is that usual law $I = \frac{1}{R}\frac{d\Phi}{dt}$ at $R \to 0$ brings to $\frac{d\Phi}{dt} = 0$ as current cannot be infinite, Then $\Phi = \text{const},$ i.e., the stream of magnetic induction remains constant — the theorem of preservation of a magnetic flux. In ours the received result does not demand the infinitesimal resistance (superconductivity).

Some additional remarks. Let's make, in the conclusion several important, in our opinion, remarks. The assumption of constancy in time of superficial and volume currents means or existence in the system of adjustable sources of currents that it is quite difficult to consider, or acceptance of assumption about ideality of the considered magnetic. The ideality is understood as very small resistance here. In this sense the stated material begins to be crossed with standard statement of introduction to the elementary theory of superconductivity. The result of the address to zero induction of magnetic field of the ideal conductor received by us closely is joined to Meissner — Ochsenfeld effect [7, 8]. However, at usual statement of this effect in the majority of courses of superconductivity draw a case of the ideal conductor and a case of superconductor nearby. And to drawings it is specified in comments that in the first case the field remains, and “real superconductors conduct differently”. According to conclusions, from the work offered by us in both cases of the field is not present inside. After shutdown of the external field in the thickness of the ideal conductor there are not fading currents that as shown above, also as well as in effect with superconductor leads to zeroing of magnetic field. Thus, artificially created difference between the ideal conductor and superconductor cleans up. Therefore, the illustration of this phenomenon passing from one book into another on which power lines in a body are specified raises at us some doubts.

Let’s note still some circumstances. As is well-known in 1935 F. London and G. London [9, 10] suggested that the ratio received from Maxwell’s equations for medium with a small resistance and which was considered as fair for $\dot{\vec{B}}$: $\Delta \dot{\vec{B}} - \frac{1}{\lambda^2}\dot{\vec{B}} = 0,$ where $\lambda^2 = \frac{m}{n\mu_0 e^2}$ is London’s length, it is necessary to consider the equation for $\vec{B},$ i.e., from equality to zero derivative $\frac{\partial}{\partial t}\left(\Delta \vec{B} - \frac{1}{\lambda^2}\vec{B}\right) = 0$ the illogical assumption of equality to zero function was made mathematically. Then, taking into account the solution of this equation: $\vec{B} = \vec{B}_0 \exp\left(-\frac{z}{\lambda}\right)$ (a one-dimensional case) Meissner effect appears. Though this theory has no strict justification and is mathematically incorrect, it became standard. At the same time from the London’s equation follows some characteristic thickness λ, in which the field exponential decreases.

It is important to note that many researchers perfectly understand all incorrectness of a conclusion of the London’s equations “…but for our purposes it is sufficient to consider it as an intuitive hypothesis fully justified by its success” [11, p. 4].

It is interesting to note that the semi-intuitive explanation of the considered phenomenon was given by R.E. Peierls [12, p. 145]: “What has gone wrong is that we have assumed we are dealing with a number of complete circular orbits. In any finite volume, however, electrons near the boundary cannot complete the circle. In addition to a number of complete circles, we must then consider also some circular arcs belonging to all those electrons whose orbits intersect the wall. These arcs amount together to a surface current which circles the volume in a sense opposite to that of the individual electron orbit, and which can easily be shown to cancel the effect of the complete orbits.”

In our model, at first sight, there is no depth of penetration of the field. Actually, if to consider that superficial current it that other as the current distributed in a thin, but final blanket and then thickness of this layer can be associated with London's length. Despite the seeming similarity with Meissner effect about expression of magnetic field, and with it and currents on a surface, it is about different things. The external field created, for example, by the toroidal coil (see Fig. 3), probably, appears in classical statement whereas in our case the fields created by own currents are considered.

The second problem is that consideration of so-called molecular currents which contribution could lead to creation of constant magnetic field with spontaneous polarization is excluded from all given constructions. However, as it was shown in [13] it is not possible within a complex form of the equations of Maxwell at least purely formally to bring together currents of conductivity and molecular currents, understanding, at the same time, that the magnetism nature, naturally general, i.e., exists physical difficulty of unambiguous division of the total current flowing in this situation on current of conductivity and molecular current of magnetization. Let's notice that considered a question, close on subject, in a series of Schwinger's works [14–17]. However, in his treatment both molecular, and polarizing currents dropped out of consideration, and at the same time the equations registered, apparently, for a vacuum and therefore characteristics of the environment did not enter that in our opinion it seems not absolutely correct.

Besides from a condition $0 = \vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right)$ and $\vec{M} = \vec{M}_0 + \chi_m \vec{H}$ the diamagnetic character of the considered phenomenon follows: magnetic susceptibility $\chi_m = -1$ (accounting of spontaneous conductivity — permanent magnets — was not carried out: $\vec{M}_0 = 0$). It is clear, that infinitely big these sizes: $\vec{M} = -\vec{H}$ cannot be and therefore, probably, as well as in case of superconductors of the first sort at some H_k (for not absolutely clear reasons) this dependence collapses and the diamagnetic moment of a magnetic addresses in zero.

The received result managed to be applied to cases of a superconducting sphere [18, 19] and the cylindrical conductor [20], and results at the same time turned out excellent from standard: magnetic induction on a surface (more precisely on a thickness commensurable with a London's depth) experiences jump [21–23].

Translated by Authors

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