UDC 551.511.31 DOI: 10.18698/1812-3368-2020-4-89-102

EVALUATING POSSIBILITY OF REGISTERING SCATTERED GRAVITATIONAL RADIATION ON WORMHOLES

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Abstract

Possibility of experimental registration of gravitational radiation scattered on wormholes was evaluated. Scattered radiation registration could become the experimental evidence of the wormhole gas theory explaining the dark matter nature. The simplest model of the traversable static spherically symmetric wormhole was used, which is the limiting case for the Bronnikov — Ellis wormhole. Equations for gravitational wave against the background of non-empty curved space–time were obtained in the gauge, where the trace of a gravitational wave is not equal to zero. It is shown that equation on the trace is reduced to the Klein — Gordon — Fock equation. Explicit expressions were obtained for the gravitational wave trace scattering cross section on a wormhole. It was assumed that the gravitational wave amplitude order was equal to its trace order, numerical simulation was carried out, and scattered gravitational radiation intensity and amplitude from wormholes on Earth were estimated. In the multiverse case, when the wormhole throat was leading to another universe, conclusion was made that it was currently impossible to register radiation scattered by wormholes taking into account the *LIGO/VIRGO* detector sensitivity

Keywords

Dark matter, gravitational waves, wormholes, registration, scattering

Received 23.09.2019
Accepted 23.12.2019

Introduction. For the first time in 1937, F. Zwicky found that mass in clusters of galaxies determined by matter luminosity was not coinciding with the virial mass determined by the galaxy motion velocities [1, 2]. Since then, it was recognized that the largest contribution to the matter density (excluding dark energy) in the Universe is made by the non-baryonic form of matter, i.e., dark matter. Studying the properties of dark matter plays a fundamental role in the development of cosmology. However, apart from the phenomenological properties of dark matter, nothing is known about its nature at present [3]. Elementary particle physics proposes various hypotheses about the dark matter particles (axions, neutralino, etc.), but so far it is only been possible to establish restrictions on such particles' experimentally (DAMA, GLAST, etc.) [4]. In connection with difficulties encountered by particle physics trying to explain the dark matter nature, it is advisable to study the alternative hypotheses. Thus, in [3, 5–8] the theory of dark matter as a cosmological wormhole gas was proposed: besides, effects of the electromagnetic radiation scattering on wormholes were predicted.

In connection with the rapid development of gravitational wave and multichannel astronomy [9], it is of great interest to study interaction between gravitational waves and wormhole gas. When interacting with wormhole gas (generating halo in vicinity of the Milky Way galaxy), alteration in the gravitational wave properties is expected; therefore, it is possible to experimentally check the hypothesis on the wormholes' presence in the halo.

Study objective is to obtain estimates of the gravitational wave scattering integral and differential cross sections on a spherically symmetric wormhole and the amplitude of scattered radiation from the wormhole halo on Earth.

Wormhole model. Wormholes' evolution depends on their structure. Spherical wormholes are unstable and collapse rapidly. They could only be stabilized by presence of the exotic matter violating the energy conditions. Less symmetrical wormholes could stably exist even without exotic matter, but they have a complex throat structure [10]. Given that wormholes have arbitrary orientation in space, the following could be assumed: due to averaging over orientations, the throat structure acquires spherical symmetry properties. This makes it possible to consider spherically symmetric wormholes in the first order.

To study scattering on wormhole, a model of the simplest spherically symmetric static wormhole will be used, which has the following metric [8]:

$$ds^2 = c^2dt^2 - K\left(r\right)^2\left(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right). \quad (1)$$

Here $K(r)$ is the conformal factor,

$$K\left(r\right) = 1 + H\left(a - r\right)\left(\left(\frac{a}{r}\right)^2 - 1\right), \quad (2)$$

where $H(r)$ is the Heaviside step function; a is the wormhole parameter. Such a wormhole has the throat length tending to zero. At $r > a$, the space is flat. Using the coordinate transformation $\tilde{r} = a^2 / r$, metric (1), (2) in the $r < a$

region could be reduced to the Minkowski metric form. Thus, the described wormhole appears to be the two separated Minkowski flat spaces (two flat universes) glued along the $S^2 \times R$ cylinder surface (direct product of time axis and spatial sphere with center in $r = 0$ and $r = a$ radius). Such wormhole is a limiting case (with the zero throat length) for the Bronnikov — Ellis wormhole with the conformal multiplier of

$$K(r) = 1 + \frac{a^2}{r^2}.$$

Wormhole scalar curvature (1), (2):

$$R^0(r) = \frac{16}{a}\delta(a - r), \tag{3}$$

where $\delta(a - r)$ is the Dirac delta function.

Important estimates of the wormhole gas parameters, which will be used in the subsequent simulation, are provided in [8]:

– $n \sim 10^{-60}$ m^{-3} wormhole density (dark matter appears on scales of 1–5 kpc);

– wormhole parameter is $a \sim (1 - 100) \cdot 10^{-3} R_{\odot}$, $R_{\odot}$ is the Sun radius.

Gravitational wave against the wormhole metric background. We are interested in gravitational wave scattering on a wormhole; therefore, it is necessary to obtain equation for the $h_{\mu\nu}$ gravitational wave against the background of curved nonempty space–time with the $g^0_{\mu\nu}$ metric tensor (Greek indices μ, ν, δ, γ, etc.) will take the values of 0, 1, 2, 3, and, if in any expression the same letter in the index designation appears both above and below, summation over all the values is taking place that could be acquired by the index. Let us suppose that gravitational wave is weak and is not perturbing energy–momentum tensor of the wormhole $T_{\mu\nu}$:

$$|h_{\mu\nu}| \ll |g^0_{\mu\nu}|, \ |\partial_\rho h_{\mu\nu}| \ll |\partial_\rho g^0_{\mu\nu}|, \ \delta T_{\mu\nu} = 0,$$

where $\partial_\gamma \equiv \dfrac{\partial}{\partial x^\gamma}$.

If a gravitational wave is missing, the $g^0_{\mu\nu}$ tensor satisfies Einstein equation:

$$R^0_{\mu\nu} = k\left(T_{\mu\nu} - \frac{1}{2} g^0_{\mu\nu} T\right), \tag{4}$$

where $R^0_{\mu\nu}$ is the Ricci tensor for the background metric of $g^0_{\mu\nu}$; $k = 8\pi G / c^4$, G are the gravitational constants, c is the speed of light; T is the energy trace of the wormhole energy–momentum tensor. General metric tensor at the gravitational wave trespassing will have the following form:

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}.$$

Gravitational wave smallness makes it possible to write down the general Ricci tensor as:

$$R_{\mu\nu} = R^0_{\mu\nu} + R^1_{\mu\nu}.$$

Here $R^1_{\mu\nu}$ is the first correction calculated by linearizing general expression for the Ricci tensor with respect to $h_{\mu\nu}$ and its derivatives,

$$R^1_{\mu\nu} = \frac{1}{2}(D_\mu D_\gamma h^\mu_\nu + D_\mu D_\nu h^\mu_\gamma - D_\gamma D_\nu h - D_\alpha D^\alpha h_{\nu\gamma}),$$

where D_ν are the covariant derivatives. The $g_{\mu\nu}$ general metric tensor satisfies the Einstein equations with the $R_{\mu\nu}$ tensor:

$$R^0_{\mu\nu} + R^1_{\mu\nu} = k\left(T_{\mu\nu} - \frac{1}{2}(g^0_{\mu\nu} + h_{\mu\nu})T\right). \tag{5}$$

Raising one contravariant index in (4) and contracting a pair of indices, the following is obtained:

$$R^0 = -kT. \tag{6}$$

Subtracting (4) from (5) and taking into account (6), the following equation is written down for a gravitational wave in the wormhole area:

$$D_\mu D_\gamma h^\mu_\nu + D_\mu D_\nu h^\mu_\gamma - D_\gamma D_\nu h - D_\alpha D^\alpha h_{\nu\gamma} - h_{\nu\gamma}R^0 = 0. \tag{7}$$

Introducing the auxiliary tensor

$$\gamma_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} g^0_{\mu\nu} h$$

and taking into consideration the identity for the second covariant derivative ($R^\rho_{\gamma\nu\mu}$ is Riemann tensor)

$$D_\mu D_\nu \gamma^\mu_\gamma = D_\nu D_\mu \gamma^\mu_\gamma + \gamma^\mu_\rho R^\rho_{\gamma\nu\mu} + \gamma_{\rho\gamma} R^{\rho\mu}_{\nu\mu},$$

in the $D_\mu \gamma^\mu_\nu = 0$ Lorentz gauge after algebraic transformations from (7), the following is obtained (δ^ν_γ is Kronecker symbol):

$$\gamma^{\mu\rho}\left(-R^\nu_{\rho\gamma\mu} - R^\nu_{\mu\gamma\rho}\right) + \gamma^\nu_\rho R^\rho_\gamma + \gamma_{\gamma\rho} R^{\rho\nu} -$$

$$- \left(D_\alpha D^\alpha \gamma^\nu_\gamma + R^0 \gamma^\nu_\gamma\right) + \frac{1}{2}\delta^\nu_\gamma \left(D_\alpha D^\alpha \gamma + R^0 \gamma\right) = 0. \tag{8}$$

If it is necessary to study alterations in the gravitational wave polarization upon scattering on a relativistic object, then it is required to solve equation (8). This is a complex mathematical problem associated with the use, for example, of tensor spherical harmonics [11], or the Newman — Penrose tetrad formalism [12, 13]. This work is only interested in the question of estimating the scattering cross section on a wormhole (1), (2) for subsequent calculation of the total amplitude of gravitational waves scattered on the wormhole halo. Therefore, it is quite possible to pass from the system of equations (8) to a simpler scalar equation on the γ gravitational wave trace, since $|\gamma| \sim |\gamma_{\nu\gamma}|$. It should be noted that the $\gamma = 0$ requirement is an additional gauge condition imposed on the metric perturbation. In the case under consideration, such gauge is replaced by condition of the $\gamma_{\mu\nu} = 0$ type at $\mu = \nu = 1$. When contracting (8) all pairs of indices, the following is obtained:

$$D_\alpha D^\alpha \gamma + R^0 \gamma = 0,$$

where the first term is the d'Alembert operator generalized for the arbitrary space–time. Using the expression for the d'Alambertian and the expression (3) for scalar curvature, the Klein — Gordon — Fock equation on the gravitational wave trace in the wormhole area is written down (1), (2):

$$\frac{1}{\sqrt{-g^0}} \frac{\partial}{\partial x^\nu} \left(\sqrt{-g^0}\, g^{0|\mu\nu} \frac{\partial \gamma}{\partial x^\mu} \right) + \frac{16}{a} \delta(a - r) \gamma = 0. \tag{9}$$

Here g^0 is the determinant of the wormhole metric tensor $g^0 =$ $= -r^4 K^6(r) \sin^2\theta$ (1), (2).

Klein — Gordon — Fock equation solution. To find the scattering cross section for a gravitational wave on the wormhole (1), (2), it is necessary to solve equation (9) imposing matching conditions on sphere $r = a$. Suppose that in the $r > a$ universe there is a plane harmonic gravitational wave propagating along the z axis. Then its trace is

$$\gamma \sim \varphi_0 = e^{ikz - i\omega t},$$

where ω is the circular frequency, and $k = \omega / c$ is the wave number. The problem to be solved is stationary, then in addition to the $r > a$ incident wave in the universe, there exists simultaneously a wormhole reflected (scattered) wave with the $\gamma \sim \varphi_{out}$ trace. In the $r < a$ universe, there exists the $\gamma \sim \varphi_{in}$ wave passing through the wormhole (hereinafter, gravitational wave shall mean its trace). Type of the φ_{out}, φ_{in} functions will be obtained below. Scattering is assumed to be elastic; therefore, $\varphi_{in}, \varphi_{out} \sim e^{-i\omega t}$.

Using the variable separation method

$$\varphi\left(t,r,\theta,\phi\right)=R(r)Y\left(\theta,\phi\right)e^{-i\omega t},$$

general solution of equation (8) is obtained. In the $r>a$ region, space is flat and solution has the following form:

$$\varphi\left(t,r,\theta,\phi\right)=\sum_{l=0,\ l\le m\le l}^{l=\infty} C_{lm}Y_{lm}\left(\theta,\phi\right)j_l(kr)e^{-i\omega t}, \tag{10}$$

where C_{lm} are the constants; $Y_{lm}\left(\theta,\phi\right)$ are the spherical harmonics; $j_l(kr)$ are the Bessel functions of the first order. Space in the $r<a$ region is also flat (in reference system with the $\tilde{r}$ radial coordinate), and solution follows:

$$\varphi\left(t,\tilde{r},\theta,\phi\right)=\sum_{l=0,\ l\le m\le l}^{l=\infty} C_{lm}Y_{lm}\left(\theta,\phi\right)j_l(k\tilde{r})e^{-i\omega t}. \tag{11}$$

It is convenient to expand the incident plane wave in Legendre polynomials:

$$\varphi_0=e^{ikz-i\omega t}=e^{ikr\cos\theta-i\omega t}=\sum_{l=0}^{l=\infty}\left(2l+1\right)i^l j_l\left(kr\right)P_l\left(\cos\theta\right)e^{-i\omega t}.$$

Expression for the incident wave is not containing angle ϕ therefore, the problem contains axial symmetry in regard to axis z. Thus, it is necessary to accept $m\equiv 0$ in (10) and (11):

$$Y_{lm}\left(\theta,\phi\right)|_{m=0}=\left(-1\right)^m e^{im\phi}\sqrt{\frac{2l+1}{4\pi}\frac{\left(l-m\right)!}{\left(l+m\right)!}}P_l^m\left(\cos\theta\right)|_{m=0}=\sqrt{\frac{2l+1}{4\pi}}P_l\left(\cos\theta\right).$$

Here $P_l^m\left(\cos\theta\right)$ are the associated Legendre polynomials. In addition, the φ_{in}, φ_{out} waves should be outgoing (φ_{out} is an outgoing wave in reference system with the $\tilde{r}$ radial coordinate). Considering that

$$j_l\left(kr\right)=h_l^+\left(kr\right)+h_l^-\left(kr\right) \tag{12}$$

and the problem is axially symmetric, let us write down expressions for waves with the C_l^{in} and C_l^{out}, unknown constants, which should be found from the boundary conditions:

$$\varphi_0=\sum_{l=0}^{l=\infty}\left(2l+1\right)i^l j_l\left(kr\right)P_l\left(\cos\theta\right)e^{-i\omega t}; \tag{13}$$

$$\varphi_{in}=\sum_{l=0}^{l=\infty}C_l^{in}\sqrt{\frac{2l+1}{4\pi}}P_l\left(\cos\theta\right)h_l^+\left(k\tilde{r}\right)e^{-i\omega t}; \tag{14}$$

$$\varphi_{out} = \sum_{l=0}^{l=\infty} C_l^{out} \sqrt{\frac{2l+1}{4\pi}} P_l(\cos\theta) h_l^+(kr) e^{-i\omega t}. \quad (15)$$

In (12), $h_l^+(kr)$ is the Hankel function of the first order describing a diverging wave; $h_l^-(kr)$ is the Hankel function of the second order describing a converging wave.

Boundary conditions should be set at the boundary of two universes. First, wave trace continuity at the boundary is required:

$$\left(\varphi_0^l + \varphi_{out}^l\right)|_{r\to a+} = \varphi_{in}^l |_{r\to a-} . \quad (16)$$

Second, let us find condition for the derivative to be continuous. When separating variables, the $R(r)$ radial part satisfies the following equation:

$$k^2 r^2 R\left(1+\left(-1+\frac{a^2}{r^2}\right)H(a-r)\right)^3 - l(1+l)\left(1+\left(-1+\frac{a^2}{r^2}\right)H(a-r)\right)R -$$
$$-r^2\frac{16}{a}\delta(a-r)\left(1+\left(-1+\frac{a^2}{r^2}\right)H(a-r)\right)^3 R +$$
$$+r^2\left(\left(1-\frac{a^2}{r^2}\right)\delta(a-r)-\frac{2a^2 H(a-r)}{r^3}\right)R_r' +$$
$$+\left(1+\left(-1+\frac{a^2}{r^2}\right)H(a-r)\right)\left(r^2 R_r'\right)_r' = 0.$$

Integrating this equation over the infinitesimal neighborhood of point $r = a$, the following expression is obtained:

$$\lim_{\varepsilon\to 0}\int_{a-\varepsilon}^{a+\varepsilon} \ldots dr = -8R(a) + a^2\left(R_r'|_{r\to a+} - R_r'|_{\to a-}\right) = 0, \quad (17)$$

where $R_r'|_{r\to a+}$ is the derivative at the $r = a$ point of the wave radial part ${\varphi_0}^l + {\varphi_{out}}^l$; $R_r'|_{\to a-}$ is the derivative at the $r = a$ point of the wave part ${\varphi_{in}}^l$.

Solving the system of equations (16) and (17), the following constants are determined:

$$C_l^{in} =$$
$$= aki^l\sqrt{2\pi l+\pi}\left(\left(h_{l+1}^+(ak) - h_{l-1}^+(ak)\right)h_l^+(ak) + h_l^+(ak)\left(h_{l-1}^-(ak) - h_{l+1}^-(ak)\right)\right)/$$
$$/\left(-h_l^+(ak)\left(akh_{l-1}^+(ak) - 17h_l^+(ak) - akh_{l+1}^+(ak)\right)\right);$$

$$C_l^{out} = i^l \sqrt{2\pi l + \pi} \left(-h_l^+ (ak) \left(2akh_{l+1}^+ (ak) - akh_{l-1}^- (ak) + 34 h_l^- (ak) + \right.\right.$$

$$\left. + akh_{l+1}^- (ak) \right) - akh_{l+1}^+ (ak) h_l^- (ak) +$$

$$\left. + akh_{l-1}^+ (ak) \left(2h_l^+ (ak) + h_l^- (ak) \right) - 34 h_l^+ (ak)^2 \right) / \left(h_l^+ (ak) \times \right.$$

$$\left. \times \left(-akh_{l-1}^+ (ak) + 17 h_l^+ (ak) + akh_{l+1}^+ (ak) \right) \right).$$

Scattering cross section. When $r \to \infty$, wave trace in the $r > a$ region could be represented as follows (time part is omitted):

$$\varphi_0 + \varphi_{out} = e^{ikz} + A(\theta) \frac{e^{ikr}}{r},$$

where $A(\theta)$ is the scattering amplitude. Differential scattering cross section is $\sigma(\theta) = A(\theta) A^* (\theta)$, $*$ is the complex conjugation, integral cross section

$$\sigma = \int_0^{2\pi} d\phi \int_0^{\pi} \sigma(\theta) \sin\theta \, d\theta = 2\pi \int_0^{\pi} \sigma(\theta) \sin\theta \, d\theta.$$

Given (13), (15) and asymptotics

$$\lim_{r \to \infty} h_l^+ (kr) = (-1)^{l+1} \frac{e^{ikr}}{kr},$$

the following expression for the scattering amplitude is obtained:

$$A(\theta) = \sum_{l=0}^{l=\infty} (-1)^{l+1} C_l^{out} \sqrt{\frac{2l+1}{4\pi}} \frac{P_l(\cos\theta)}{k}.$$

Specific differential cross section for scattering gravitational wave with a frequency of ν = 250 Hz on a wormhole is shown in Fig. 1. Scattering mainly occurs forward, which corresponds to diffraction on an obstacle in the form of a wormhole.

Integral scattering cross section dependence for two gravitational waves frequencies from characteristic space sources ($\nu \sim 100$ Hz, rotation and collision of objects in a binary system, neutron stars, black holes; $\nu \sim 1{,}000$ Hz, gravitational collapse of a star) is presented in Fig. 2. Integral cross section evaluation within characteristic frequencies could be performed as:

$$\sigma \sim 10 \sigma_{sphere} = 10 \pi a^2,$$

where σ_{sphere} is the elastic scattering cross section on a solid sphere.

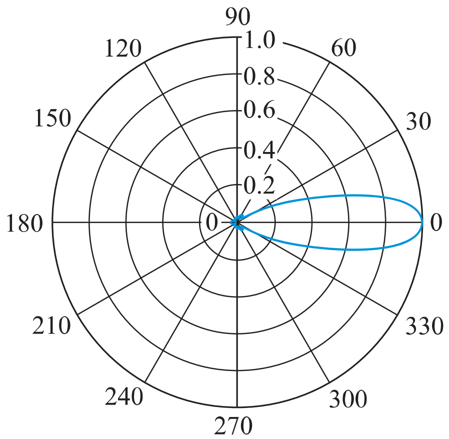

Fig. 1. Specific differential cross section of the gravitational wave scattering (ν = 250 Hz, a = 10^6 m) on a wormhole (1), (2)

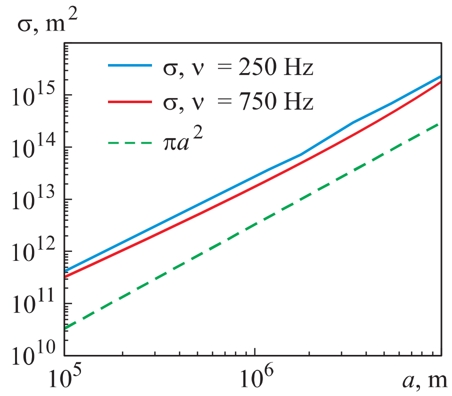

Fig. 2. Integral cross section dependence on the a parameter for two characteristic frequencies of the gravitational wave (in addition, scattering cross section on a solid sphere is shown)

Scalar wave scattering cross section on the Bronnikov — Ellis wormhole in the low-frequency limit was obtained in [14]: $\sigma \approx 64a^2$ which in order coincides with the estimate obtained here. Thus, when studying the low-frequency gravitational waves scattering on the Bronnikov — Ellis wormhole, using the simplified model (1), (2) was justified.

Scattered waves amplitude numerical calculation. Knowing the gravitational wave scattering cross section on a wormhole, it is possible to estimate the wormhole gas influence manifesting in the form of the Milky Way dark matter halo on the order of intensity and amplitude of gravitational waves registered by Earth detectors.

Let us calculate intensity order of intensity of a gravitational wave scattered on a single i-th wormhole and measured on the Earth (Fig. 3). The h' scattered wave

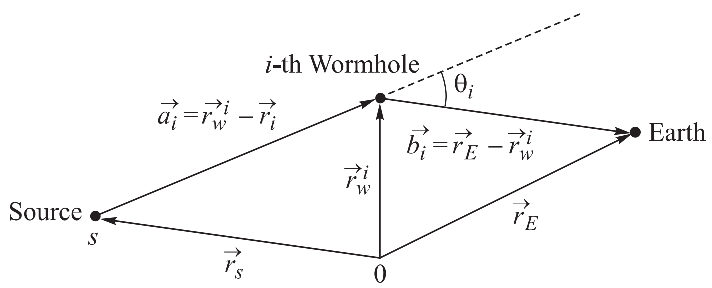

Fig. 3. Scheme for calculating the scattered gravitational wave amplitude on Earth

trace at the high r values from the wormhole is described by the following function:

$$h' = A(\theta)\frac{e^{ikr}}{r}e^{-i\omega t}.$$

Gravitational wave intensity with the h trace could be estimated as [15]:

$$I \sim cW \sim c\frac{c^4}{G}\left|\partial_0 h\right|^2 = \frac{\omega^2 c^3}{G}\left|h\right|^2, \qquad (18)$$

where W is the gravitational wave energy density. Ratio between scattered (h') and plane incident (h_0) waves intensity is obtained:

$$\frac{I'}{I_0} = \frac{\left|h'\right|^2}{\left|h_0\right|^2} = \frac{\left|A(\theta)\frac{e^{ikr}}{r}e^{-i\omega t}\right|^2}{\left|e^{ikz}e^{-i\omega t}\right|^2} = \frac{\sigma(\theta_i)}{\left|\vec{b}_i\right|^2}. \qquad (19)$$

Here θ_i is the scattering angle; $\left|\vec{b}_i\right|$ is the distance from the wormhole to the Earth. In the framework of the linearized theory, amplitude order for gravitational wave from a small dynamic source with the characteristic size L, mass M and rotation frequency ω could be estimated as follows:

$$\left|h_0\right| \sim \left|h_{\mu\nu}\right| \sim \frac{GM}{c^4\left|\vec{a}_i\right|}L^2\omega^2, \qquad (20)$$

where $\left|\vec{a}_i\right|$ is the distance from a wormhole to the source. Then, from (20) and taking into account that a plane wave falls on a wormhole (spherical wave from a source degenerates into a plane wave at large distances), the following expression is obtained:

$$I_0 \sim \frac{\omega^2 c^3}{G}\left|h_0\right|^2 = \frac{\omega^2 c^3}{G}\left(\frac{GM}{c^4\left|\vec{a}_i\right|}L^2\omega^2\right)^2.$$

Thus, from (19) intensity of a wave scattered by a single wormhole on Earth could be estimated by the following expression:

$$I' \sim I_0\frac{\sigma(\theta_i)}{\left|\vec{b}_i\right|^2} = \frac{\omega^6 GM^2L^4}{c^5}\frac{\sigma(\theta_i)}{\left|\vec{a}_i\right|^2\left|\vec{b}_i\right|^2}.$$

Aggregate intensity of scattered waves from all the N wormholes is:

$$I' \sim \sum_{i=1}^{N}\frac{\omega^6 GM^2L^4}{c^5}\frac{\sigma(\theta_i)}{\left|\vec{a}_i\right|^2\left|\vec{b}_i\right|^2}; \qquad (21)$$

$$\theta_i = \arccos \frac{\vec{a}_i \vec{b}_i}{\left|\vec{a}_i\right|\left|\vec{b}_i\right|}. \quad (22)$$

Simulation was conducted to calculate the aggregate intensity of scattered waves on the Earth according to (21), (22). Problem geometry is presented in Fig. 4. Source parameters were selected similar to source parameters of the first registered gravitational waves [16], i.e., of the binary black hole. Gravitational waves from this source are emitted at a double orbital frequency, but this does not affect the order of intensity. To simplify the problem, let us suppose that the source is located in the Milky Way galactic disk plane (*XY* plane). Reference point is the spherical halo center, the Earth and the source are located on the *X* axis. In addition, only contribution from the first scattering should be considered; so, each subsequent scattering reduces intensity due to multiplying by factor $\sim \sigma(\theta_i) / R_{halo}^2 \ll 1$, i.e., contribution from waves scattered two or more times is much less than contribution from the first scattering.

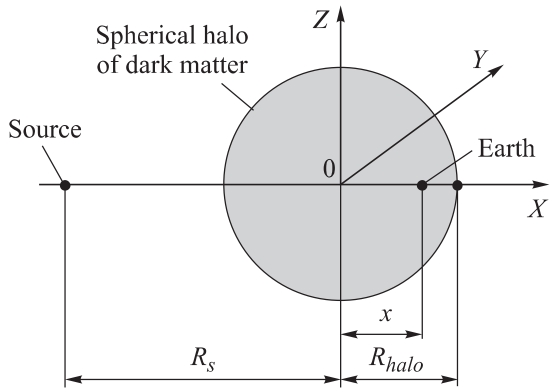

Fig. 4. Simulation geometry (length scale not respected)

Uniform distribution of wormholes over the galactic halo volume was simulated in a probabilistic manner (varying *n* density). Differential and integral cross sections values (for the *a* wormhole corresponding parameter and the gravitational wave frequency) obtained in this work were used in simulation. Simulation parameters are presented below (1 pc $\approx$ 3.26 ly $\approx$ $3.09 \cdot 10^{16}$ m):

Density n, pc^{-3} .... 10^{-10}–10^{-8} in increments of $5 \cdot 10^{-7}$
Black holes total mass M, ... 10^{32}
ω, rad/s 2,500
Distance between black holes centers at the moment of at the moment of contact of the horizons .... $L \; \frac{4GM}{c^2} (\sim 2r_g)$

R_s, pc .... 10^8
a, pc .... 10^7
x_E, pc .... $8.5 \cdot 10^3$
R_{halo}, pc .... $5 \cdot 10^4$

Estimate of the gravitational wave amplitude arriving directly from the source gives $|h_E| \sim 10^{-18}$. Simulation result of the scattered waves total amplitude is presented in Fig. 5 (amplitude and intensity are connected by (18)). Analytical curve is constructed using the least squares method in the following form: $|h'| \sim 3 \cdot 10^{-26} \sqrt{n}$.

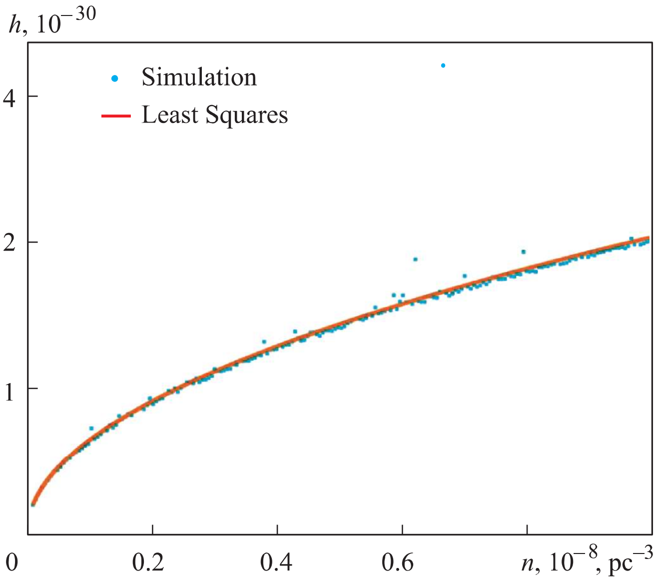

Fig. 5. Result of simulating the scattered waves total amplitude on Earth

In the studied density range, $|h'| \sim 10^{-30}$ is much smaller than $|h_E|$. At present, sensitivity of *LIGO/VIRGO* gravitational detectors does not exceed 10^{-23} in the ~ 100 Hz frequency band. This leads to a conclusion that in the near future registering diffraction of the gravitational wave signal on wormhole gas (at least within the framework of the considered model) would be impossible.

Conclusion. Experimental possibility of registering gravitational radiation scattered by wormhole gas on Earth was studied. For this, scattering of a plane gravitational wave on static and spherically-symmetric wormhole was considered. Tensor differential equations for a gravitational wave in the wormhole area were reduced to the Klein — Gordon — Fock equation on the gravitational wave trace, which was solved. Such a method made it possible to estimate the gravitational wave scattering cross sections on a wormhole, but not alteration in the gravitational wave polarization.

It was found that scattering was taking place mainly forward, which corresponded to diffraction by an obstacle. It was supposed that the wormhole troats were having a second exit to another universe [17] or to a fairly remote area of our Universe. Integral scattering cross section in the region of interest in regard to the wormhole parameter value could be estimated with accuracy to within an order of magnitude as $\sigma \sim 10\pi a^2$. Integral cross section was decreasing with an increase in the gravitational wave frequency.

Numerical calculation demonstrated that the scattered gravitational radiation amplitude was growing in a root way with an increase in the wormholes' density in the Milky Way halo. In the region of interest for density values, it constituted $|h'| \sim 10^{-30}$. This was significantly less than the amplitude that *LIGO/VIGRO* collaborations could register. Thus, registration of gravitational waves diffraction by the wormhole gas seems to be impossible at present.

Translated by K. Zykova

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Please cite this article as:

Kirillov A.A., Krichevskiy D.P. Evaluating possibility of registering scattered gravitational radiation on wormholes. *Herald of the Bauman Moscow State Technical University, Series Natural Sciences*, 2020, no. 4 (91), pp. 89–102.
DOI: https://doi.org/10.18698/1812-3368-2020-4-89-102