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ANALYTICAL STUDY OF NONSTATIONARY MODES IN RECUPERATIVE HEAT EXCHANGERS

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Abstract

Recuperative heat exchanger transient operation modes during the start-up were considered in order to identify the time for establishing the stationary mode. This is important in carrying out technological processes that require constancy in values of certain parameters ensuring both product quality and process safety. The research was carried out using the analytical method for direct-flow and counter-flow heat exchangers. It was demonstrated that stationary state establishment in the direct-flow heat exchangers occurs immediately after the heat carrier gets into the apparatus. It should be noted that the entire apparatus reaches the stationary mode, when the slower heat carrier arrives at the apparatus output section. In case of a heat exchanger with the heat carrier counter-flow, it was found out that at the moment of the less heated heat carrier appearing at the apparatus output section, it was having the highest temperature. Then the temperature was decreasing, and after passing its minimum was beginning to oscillate along a curve with the damping amplitude. In the case under consideration, the stationary process started, when the dimensionless time value was $\phi \geq 0.5$. The indicated solution was obtained under assumption that thermal and physical characteristics were constant in time and space. It was assumed that total heat capacity of the heat exchanger heat transferring wall was infinitesimal. This assumption is valid with an error of up to 1 % at $\mathrm{Fo} \geq 100$, which is the case in most practical cases. For apparatuses under study, a formula was also obtained for the time required to reach the stationary state

Keywords

Analytical studies, direct-flow heat exchanger, counter-flow heat exchanger, convective heat exchange, turbulent flow, system of convective heat exchange equations, Bessel function

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Introduction. In some cases, when carrying out thermal technological processes, it is practically required to maintain the values of a number of technological parameters at a certain level and to ensure product quality or process safety [1]. Under these circumstances, stabilization time period of the technological process parameters becomes of great importance. Such requirements arise, when solutions are heated in technological equipment, thermostats, heat accumulators, etc. It is of decisive importance to estimate the time it takes to reach the stationary state, when heat exchangers (HE) are enabled being an integral part of these processes.

Issues related to analytical solution of heat exchange nonstationary problems were considered in many works, for example, in [2–8]. Such problems include the processes of heating and cooling various bodies under different unambiguity conditions, which significantly influences the final form of the process mathematical description. Analytical studies of transient processes in the recuperative heat exchangers were mainly carried out by numerical methods, which ultimate objective was primarily to obtain solution for a stationary state [9–13].

The purpose of the work is to consider the nonstationary heat exchange during the recuperative heat exchanger start-up and to propose a solution to this problem with the uniqueness conditions provided below.

Methods for solving the problems and accepted assumptions. To solve the problem set, let us consider the processes of heat exchange in a direct-flow and in a counter-flow HE with the following assumptions: heat exchange with the environment is missing; aggregated heat capacity of structural materials is insignificant and could be neglected; heat carrier thermal and physical properties do not depend on temperature; developed turbulent flow is taking place, therefor, the temperature over the heat carrier flow cross section is invariable.

Nonstationary heat exchange in direct-flow heat exchanger. Let us assume that at the initial moment of time heat carriers appear at the apparatus input sections simultaneously, and their temperature at the input remains constant in time. Thus, heat exchange between both heat carriers during the apparatus filling could occur in the area with slower heat carrier [14]. The following condition should be met $x \le w_{\min}\tau$, where $w_{\min}$ is the minimum heat carrier speed. In this case, the system of convective heat exchange equations has the following form [15]:

$$k_T\left(t_1 - t_2\right) = \rho_1 c_1 f_1\left(\frac{\partial t_1}{\partial \tau} + w_1\frac{\partial t_1}{\partial x}\right);$$
$$k_T\left(t_2 - t_1\right) = \rho_2 c_2 f_2\left(\frac{d t_2}{d \tau} + w_2\frac{d t_2}{d x}\right). \qquad (1)$$

Here k_T is the heat transfer coefficient; ρ, c, f are heat carriers specific density and heat capacity, channel cross-section area for heat carriers; $w_{1,2}$ are the heat carriers motion speed (indices "1" and "2" refer to the corresponding heat carriers); $t_{1,2}$ are the heat carrier temperatures; τ is the current time.

Let's introduce the following notations:

$$\begin{aligned}
&\frac{\rho_1 c_1 f_1}{k_T} = m_1; \quad \frac{\rho_2 c_2 f_2}{k_T} = m_2;\\
&\rho_1 c_1 f_1 w_1 = W_1; \quad \rho_2 c_2 f_2 w_2 = W_2;\\
&\frac{m_1}{m_2} = \psi_{12}; \quad \frac{m_2}{m_1} = \psi_{21};\\
&\frac{W_1}{W_2} = \Psi_{12}; \quad \frac{W_2}{W_1} = \Psi_{21};\\
&\frac{\tau}{m_1} = \tau'; \quad \frac{k_T x}{W_1} = x'.
\end{aligned} \tag{2}$$

Given the notations (2), the system of Eq. (1) takes the following form:

$$\begin{aligned}
&t_1 = t_2 + \psi_{21}\frac{\partial t_2}{\partial \tau'} + \Psi_{21}\frac{\partial t_2}{\partial x'};\\
&t_2 = t_1 + \frac{\partial t_1}{\partial \tau'} + \frac{\partial t_1}{\partial x'}.
\end{aligned} \tag{3}$$

Combining both equations of system (3), the following expression is obtained:

$$\frac{\partial^2 \vartheta_1}{\partial x'^2} + \left(1 + \psi_{21}\Psi_{12}\right)\frac{\partial^2 \vartheta_1}{\partial x' \partial \tau'} + \psi_{21}\Psi_{12}\frac{\partial^2 \vartheta_1}{\partial \tau'^2} +$$

$$+ \left(1 + \Psi_{12}\right)\frac{\partial \vartheta_1}{\partial x'} + \Psi_{12}\left(1 + \psi_{21}\right)\frac{\partial \vartheta_1}{\partial \tau'} = 0, \tag{4}$$

where

$$\vartheta_1 = \frac{t_1 - t_{1\,initial}}{t_{2\,initial} - t_{1\,initial}}.$$

Index "*initial*" denotes the initial temperature values of heat carriers 1 and 2.

It should be noted that the heat transfer between the heat carriers when filling the HE is carried out only in the section filled with the slower heat carrier. In accordance with the above, the boundary conditions could be written as follows:

$$\vartheta_1\big|_{x'=0} = 0;$$

$$\left.\frac{\partial\vartheta_1}{\partial x'}\right|_{x'=0} + \left.\frac{\partial\vartheta_1}{\partial\tau'}\right|_{x'=0} = 1.$$

The second relation follows from the second equation of system (3).

Initial condition is $x'\big|_{\tau'=0} = 0.$ Let us bring Eq. (4) to a form convenient for solving. To do this, let us introduce new independent variables:

$$\varphi = \sqrt{\Psi_{12}}\,\frac{\tau' - x'}{1-\psi_{21}\Psi_{12}};\;\; \phi = \sqrt{\Psi_{12}}\,\frac{\tau' - \psi_{21}\Psi_{12}x'}{1-\psi_{21}\Psi_{12}}. \tag{5}$$

As a result of Eq. (3) transformation, the following expression is obtained:

$$\frac{\partial^2\vartheta_1}{\partial\phi\partial\varphi} - \sqrt{\Psi_{12}}\,\frac{\partial\vartheta_1}{\partial\varphi} + \sqrt{\Psi_{21}}\,\frac{\partial\vartheta_1}{\partial\phi} = 0.$$

Solution will be sought in the following form:

$$\vartheta_1 = U(\phi,\varphi)\,e^{\alpha\phi+\beta\varphi}, \tag{6}$$

where $\alpha = \sqrt{\Psi_{12}};\; \beta = -\sqrt{\Psi_{21}} = -1/\alpha.$ Then

$$\frac{\partial^2 U}{\partial\phi\,\partial\varphi} + U = 0. \tag{7}$$

Boundary conditions:

$$U\big|_{\phi=\varphi} = 0;\;\; \left.\frac{\partial U}{\partial\varphi}\right|_{\phi=\varphi} = \frac{1}{\alpha}\,e^{-\left(\alpha-\frac{1}{\alpha}\right)\varphi}. \tag{8}$$

Eq. (7) together with the boundary conditions (8) is solved by the Riemann method [6–8]:

$$U = \frac{1}{\alpha}\int_{\phi}^{\varphi} e^{-\left(\alpha-\frac{1}{\alpha}\right)Z} I_0\left[2\sqrt{(\varphi-Z)(Z-\phi)}\right]dZ,$$

where I_0 is the zero order Bessel function.

Taking into account (6), the following expression is obtained:

$$\vartheta_1 = \frac{1}{\alpha}\int_{\phi}^{\varphi} e^{-\alpha(z-\phi)-\frac{1}{\alpha}(\varphi-\phi)} I_0\left[2\sqrt{(\varphi-Z)(Z-\phi)}\right]dZ. \tag{9}$$

Let us introduce a new variable that satisfies condition $Z = \mu + \varphi Z$, where μ is the new integration variable. In accordance with this, Eq. (9) is converted to the following form:

$$\vartheta_1 = \frac{1}{\alpha} \int\limits_0^{\varphi-\phi} e^{-\alpha\mu - \frac{1}{\alpha}(\varphi-\phi-\mu)} I_0\left[2\sqrt{(\varphi-\phi-\mu)\mu}\right] d\mu. \qquad (10)$$

It follows from Eq. (10) that the ϑ_1 value depends only on $\alpha = \sqrt{\Psi_{12}}$ and $(\varphi - \phi)$. Taking into consideration Eq. (5), the following is obtained: $(\varphi - \phi) = \alpha x'$. Therefore, heat carriers temperature in the direct-flow HE depends only on the coordinate and does not depend on time. In other words, stationary temperature regime in the HE is established immediately after the heat carriers enter the apparatus; and the temperature regime along the entire apparatus length is established, when the slower heat carrier reaches the HE output section.

On the one hand, Eq. (10) describes heat carrier temperature distribution in the apparatus at a steady state. On the other hand, temperature distribution in a direct-flow apparatus in the steady state, taking into account the new variables, is described by the following relation:

$$t_1 = t_{1\,initial} + \frac{t_{2\,initial} - t_{1\,initial}}{1+\Psi_{12}}\left[1 - e^{-x'(1+\Psi_{12})}\right]$$

or

$$\vartheta_1 = \frac{1}{1+\alpha^2}\left[1 - e^{-(\varphi-\phi)\left(\alpha+\frac{1}{\alpha}\right)}\right]. \qquad (11)$$

Solutions to Eqs. (10) and (11) for specific cases provide the same results. Thus, the following could be written down:

$$\int\limits_0^{\varphi-\phi} e^{-\alpha\mu - \frac{1}{\alpha}(\varphi-\phi-\mu)} I_0\left[2\sqrt{(\varphi-\phi-\mu)\mu}\right] d\mu =$$

$$= \frac{1}{\alpha + \frac{1}{\alpha}}\left[1 - e^{-(\varphi-\phi)\left(\alpha+\frac{1}{\alpha}\right)}\right]. \qquad (12)$$

Nonstationary heat exchange in a counter-flow heat exchanger. When considering nonstationary heat exchange during the apparatus start-up at the heat carriers counter-flow motion, the same assumptions were used as for their direct-flow motion. It was assumed that one of the heat carriers (for definiteness — the second one) moved constantly along the HE in the direction opposite to the x axis. Another heat carrier was starting to fill the heat exchanger at the $\tau_0 = 0$ moment of time, when the HE was completely filled with heat carrier 2. Heat carriers' temperature at the input was being kept constant. In this case, the heat carrier 1 core area before it left the apparatus was interacting with the heat carrier 2 flow having constant temperature of $t_{2\,initial}$ [14, 15]. Therefore, temperature field for the steady state is described by the following expression:

$$1-\vartheta_1 = \frac{\left(1-\Psi_{12}\right)e^{-x'\left(1-R_{12}\right)}}{1-\Psi_{12}e^{-x'\left(1-R_{12}\right)}}. \qquad (13)$$

In this case, the heat exchange process is described by Eqs. (3) and (4), which is the same for a direct-flow heat exchanger when reversing the sign at ψ_{12}.

Let us introduce the dimensionless temperatures:

$$\vartheta_1 = 1-\frac{t_{2\,initial}-t_1}{t_{2\,initial}-t_{1\,initial}};\quad 1-\vartheta_1 = \frac{t_{2\,initial}-t_1}{t_{2\,initial}-t_{1\,initial}};\quad \vartheta_2 = \frac{t_{2\,initial}-t_2}{t_{2\,initial}-t_{1\,initial}}.$$

Eqs. (7) and (8) will take the following form:

$$\vartheta_2 = 1-\vartheta_1+\frac{\partial\left(1-\vartheta_1\right)}{\partial x'}+\frac{\partial\left(1-\vartheta_1\right)}{\partial \tau'}; \qquad (14)$$

$$\frac{\partial^2\left(1-\vartheta_1\right)}{\partial x'^2}+\left(1-\psi_{21}\Psi_{12}\right)\frac{\partial^2\left(1-\vartheta_1\right)}{\partial x'\partial \tau'}-\psi_{21}\Psi_{12}\frac{\partial^2\left(1-\vartheta_1\right)}{\partial \tau'^2}+$$

$$+\left(1-\Psi_{12}\right)\frac{\partial\left(1-\vartheta_1\right)}{\partial x'}-\Psi_{12}\left(1+\psi_{21}\right)\frac{\partial\left(1-\vartheta_1\right)}{\partial \tau'}=0. \qquad (15)$$

Taking into consideration the accepted assumptions, boundary conditions are obtained. When filling the HE, the heat carrier 1 face area along the entire apparatus length is interacting with the heat carrier 2 flow at its constant initial temperature $t_{1\,initial}$. Thus, relation is valid for the heat carrier 1 face area, which was obtained for stationary heat transfer conditions. And in case of infinitely large value of the heat carrier 2 thermal equivalent, it could be represented as $\vartheta_1 = 1-e^{-x'}$. Hence, condition appears that should be satisfied by the solution of Eq. (15) and is written down in the form of $\left(1-\vartheta_1\right)\big|_{x=w_x\tau} = e^{-x'}$.

Heat carrier 2 temperature in the $x = w_1\tau$ section would be equal to the $t_{2\,initial}$ initial value, which in combination with Eq. (15) makes it possible to obtain the second condition:

$$\left(1-\vartheta_1\right)\Big|_{x=w_x\tau}+\left.\frac{\partial\left(1-\vartheta_1\right)}{\partial x'}\right|_{x=w_x\tau}+\left.\frac{\partial\left(1-\vartheta_1\right)}{\partial \tau'}\right|_{x=w_x\tau}=0.$$

For the HE input sections, boundary conditions could be written down as $\left(1-\vartheta_1\right)\big|_{x'=0}=1$ and with (14):

$$\left(1-\vartheta_1\right)\Big|_{x'=l'}+\left.\frac{\partial\left(1-\vartheta_1\right)}{\partial x'}\right|_{x'=l'}+\left.\frac{\partial\left(1-\vartheta_1\right)}{\partial \tau'}\right|_{x'=l'}=0.$$

Let us introduce new independent variables:

$$\phi=x';\ \ \varphi=\frac{\tau'-x'}{1+\psi_{21}\Psi_{12}}. \tag{16}$$

Eq. (15) takes the following form:

$$\frac{\partial^2\left(1-\vartheta_1\right)}{\partial\phi^2}-\frac{\partial^2\left(1-\vartheta_1\right)}{\partial\phi\,\partial\varphi}+\left(1-\Psi_{12}\right)\frac{\partial\left(1-\vartheta_1\right)}{\partial\phi}-\frac{\partial\left(1-\vartheta_1\right)}{\partial\varphi}=0. \tag{17}$$

Using substitution

$$1-\vartheta_1=Ue^{\alpha\phi+\beta\varphi}, \tag{18}$$

$$\alpha=-1;\ \ \beta=-\left(1+\Psi_{12}\right),$$

we obtain

$$\frac{\partial^2 U}{\partial\phi^2}-\frac{\partial^2 U}{\partial\phi\,\partial\varphi}+\Psi_{12}U=0.$$

Boundary conditions:

$$U\Big|_{\varphi=0}=1;\quad \left.\frac{\partial U}{\partial\phi}\right|_{\varphi=0}=0;$$

$$U\Big|_{\phi=0}=e^{-\beta\varphi};\quad \left.\frac{\partial U}{\partial\phi}\right|_{\phi=l'}=0.$$

Applying the Laplace transform in the ϕ variable, the following is determined:

$$\frac{\partial^2\overline{U}}{\partial\phi^2}-p\frac{\partial\overline{U}}{\partial\phi}+\Psi_{12}\overline{U}=0. \tag{19}$$

Boundary conditions:

$$\overline{U}\Big|_{\phi=0}=\frac{p}{p+\beta};\quad \left.\frac{\partial\overline{U}}{\partial\phi}\right|_{\phi=l'}=0. \tag{20}$$

Condition (20) characterizes the state after heat carrier 2 appears in the output section of the heat exchanger, i.e., $\tau' \geq l'$.

Solution to equation (19):

$$\overline{U} = Ae^{\frac{p+\lambda}{2}\phi} + Be^{\frac{p-\lambda}{2}\phi}, \tag{21}$$

where $\lambda = \sqrt{p^2 - 4\Psi_{12}}$.

After solving Eq. (19) with boundary conditions (20) in the form of Eq. (21) and corresponding transformations, the final expression is obtained for the U original function at the HE output:

$$U\big|_{\phi=l'} = e^{l'-\beta\varphi}\frac{(1-\Psi_{12})e^{-l'(1-\Psi_{12})}}{1-\Psi_{12}e^{-l'(1-\Psi_{12})}} + \\ +\frac{4}{m}\sum_{j=1}^{m}\left\{(1-A_j)\cos\left[\left(1+2\frac{\varphi}{l'}\right)\nu_j \operatorname{th}\mu_j\right] + \right. \\ \left. + B_j \sin\left[\left(1+2\frac{\varphi}{l'}\right)\nu_j \operatorname{th}\mu_j\right]\right\}\exp\left[-\left(1+2\frac{\varphi}{l'}\right)\mu_j \operatorname{cth}\mu_j\right]. \tag{22}$$

Hence, taking into account (18), expression for dimensionless temperature at the HE output is written down:

$$1-\vartheta_{1l} = \frac{(1-\Psi_{12})e^{-l'(1-\Psi_{12})}}{1-\Psi_{12}e^{-l'(1-\Psi_{12})}} + \frac{4}{m}e^{-l'}\sum_{j=1}^{\infty}\left\{(1-A_j)\cos\left[\left(1+2\frac{\varphi}{l'}\right)\nu_j \operatorname{th}\mu_j\right] + \right. \\ \left. + B_j \sin\left[\left(1+2\frac{\varphi}{l'}\right)\nu_j \operatorname{th}\mu_j\right]\right\}\exp\left[-\left(1+2\frac{\varphi}{l'}\right)\mu_j \operatorname{cth}\mu_j - \varphi(1+\Psi_{12})\right], \tag{23}$$

where

$$A_j = \left\{(1+b)\left[(1-\mu_j \operatorname{cth}\mu_j)^2(b-\mu_j \operatorname{cth}\mu_j) - \nu_j^2(1+\mu_j \operatorname{cth}\mu_j)\operatorname{th}^2\mu_j\right]\right\}\times \\ \times\left\{\left[(b-\mu_j \operatorname{cth}\mu_j)(1-\mu_j \operatorname{cth}\mu_j) - \nu_j^2 \operatorname{th}^2\mu_j\right]^2 + \right. \\ \left. + \nu_j^2(1+b-2\mu_j \operatorname{cth}\mu_j)^2 \operatorname{th}^2\mu_j\right\}^{-1} + \nu_j^2\left[(1+b)^2 - (m^2-1)\right]\operatorname{th}^2\mu_j \times \\ \times\left\{\left[(b-\mu_j \operatorname{cth}\mu_j)(1-\mu_j \operatorname{cth}\mu_j) - \nu_j^2 \operatorname{th}^2\mu_j\right]^2 + \right. \\ \left. + \nu_j^2(1+b-2\mu_j \operatorname{cth}\mu_j)^2 \operatorname{th}^2\mu_j\right\}^{-1};$$

$$B_j = -\nu_j \operatorname{th}\mu_j \left\{ \left[\left(1-\mu_j \operatorname{cth}\mu_j\right)^2 + \nu_j^2 \operatorname{th}^2 \mu_j + m^2 - 1 \right] (1+b) - \right.$$
$$\left. -2\mu_j \operatorname{cth}\mu_j \left(m^2-1\right) \right\} \left\{ \left[\left(b-\mu_j \operatorname{cth}\mu_j\right)\left(1-\mu_j \operatorname{cth}\mu_j\right) - \nu_j^2 \operatorname{th}^2 \mu_j \right]^2 + \right.$$
$$\left. + \nu_j^2 \left(1+b-2\mu_j \operatorname{cth}\mu_j\right)^2 \operatorname{th}^2 \mu_j \right\}^{-1}.$$

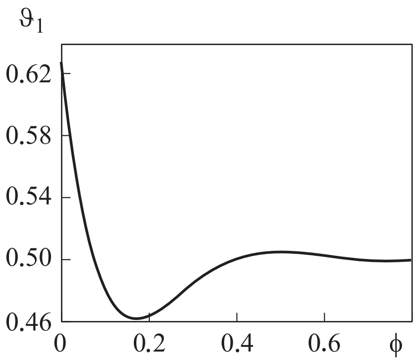
Here $m^2 = l'^2 \Psi_{12}$; $b = \dfrac{\beta}{2} l'$; μ, ν are the values that are found from transcendental equations $m \operatorname{sh}\mu \cos\nu = -\mu$, $m \operatorname{ch}\mu \sin\nu = -\nu$.

The first term of Eq. (23) is similar to Eq. (13) and reflects the steady state. The second term characterizes deviation from steady state in the transition process.

As noted, solution (23) was obtained under the assumption of thermal and physical characteristics constancy in time and space. In this regard, thermal and physical values included in the obtained solutions should be averaged over time and space.

Example. Let us provide solution to the problem for $m = 1$ and $b = -1$, which corresponds to $l' = 1$ and $\Psi_{12} = 1$. Relation (23) is significantly simplified:

$$1 - \vartheta_{1l} =$$
$$= 0.5 + 1.47 \exp(-2\varphi) \sum_{i=1}^{\infty} \exp\left(-(1+2\varphi)\mu_i \operatorname{cth}\mu_i\right) \cos\left[(1+2\varphi)\nu_i \operatorname{th}\mu_i\right]. \quad (24)$$



Dependence of the ϑ_1 dimensionless temperature on the ϕ dimensionless time at $l' = 1$ and $\Psi_{12} = 1$

Dependence of the ϑ_1 dimensionless temperature on the ϕ dimensionless time at $l' = 1$ and $\Psi_{12} = 1$ is constructed according to (24) and presented in the Figure. At the time less heated heat carrier appears at the HE output section it has the highest temperature. Then the temperature decreases and having passed the minimum begins to oscillate along a curve with damping amplitude. In the case under consideration, the stationary process occurs at $\phi \geq 0.5$.

Given Eqs. (2) and (16), the time to reach stationary state could be estimated by the following formula:

$$\tau = \left[0.5\left(1+\frac{w_1}{w_2}\right) + 1 \right] \frac{\rho_1 c_1 f_1}{k_T}. \quad (25)$$

As noted, Eq. (23) was obtained under assumption that accumulating capacity of the HE heats exchanging surface was infinitely small. This is true with an error of up to 1 % at Fo ≥ 100, which corresponds to the $\tau \geq 100\,\delta^2 \big/ \left(4a_{wall}\right)$ time (δ is the HE heats exchanging wall thickness; a_{wall} is the wall heat conductivity coefficient). In most cases, assumption on the negligibly small accumulating capacity of the wall turns out to be true.

Conclusion. To determine time required to establish the stationary mode, analytical studies of transient conditions during the recuperative heat exchangers start-up were carried out. Stationary state in the direct-flow HE occurs almost immediately after the heat carriers enter the apparatus. HE reaches the stationary mode, when the slower heat carrier gets to the apparatus output section.

When the counter-flow HE is enabled, oscillation of the heat carrier temperature is observed. Maximum temperature in the mode under study is observed with the slower heat carrier during its output from the HE. Then the temperature starts to oscillate with the damping amplitude. A formula is derived for the time to reach the stationary state.

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